Firm Choices of Wage-Setting Protocols in the Presence of Minimum Wages

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Preliminary and Incomplete. See Latest Version

Abstract

In this paper we study the formation of wages in a frictional search market where firms can choose either to bargain with workers or post non-negotiable wage offers. Workers can secure wage increases for themselves by engaging in on-the-job search and either moving to firms that offer higher wages or, when possible, leveraging an outside offer into a higher wage at the current firm. We characterize the optimal wage posting strategy of non-negotiating firms and how this decision interacts with the presence of renegotiating firms. The model has important implications for worker wage dispersion, efficiency of worker mobility decisions, and welfare. We quantitatively examine these implications by estimating the model, using data on the wages and employment spells of low-skill workers in the United States. Finally, as a policy application of the study, we assess the impact of binding minimum wages on firm bargaining strategies and worker welfare in a general equilibrium setting.

1 Introduction

Using data collected from a sample of recent hires, Hall and Krueger (2012) show that in setting initial compensation some firms specify a fixed, non-negotiable wage or salary, while other firms negotiate with the new employee over compensation levels. Approximately one-third of sample members report having bargained with their employers at the time of their initial hiring, with bargaining more likely to have occurred for more highly educated workers. In these cases, they found that their current employers had learned their compensation in earlier jobs before making the (accepted) compensation offer in the current job.

These findings suggest that employers may employ different strategies when hiring workers, with some essentially following a wage-posting paradigm, while others actively engage in bargain-

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ing. Although Hall and Krueger find evidence that there is a systematic relationship between the characteristics of the worker and the market in which they are searching on the likelihood that compensation was set through bargaining, within any class of workers or market there are cases in which wages were bargained over and others in which they were not. This heterogeneity in wage determination methods is not examined within the vast majority of partial and general equilibrium models of labor market search. In models of wage posting, employers make take-it-or-leave-it offers to applicants, which the applicant either accepts or rejects. Perhaps the most well-known models of wage posting are Albrecht and Axell (1984) and Burdett and Mortensen (1998). In these models, firms offer fixed wages to all applicants they encounter, and it is often assumed that all applicants are equally productive. In the Burdett and Mortensen model, workers of homogeneous productivity are offered different wages by ex ante identical firms. Their model produces an equilibrium wage offer distribution and steady state wage distribution that are nondegenerate, even though all workers and firms are ex ante identical.[1]

Most wage bargaining models estimated using individual-level data are based an assumption of ex ante heterogeneity in worker and/or firm productivities. Most typically, some sort of cooperative bargaining protocol is assumed, such as Nash bargaining or simply surplus division. In the cases in which on-the-job (OTJ) search is introduced, assumptions are made regarding the amount of information available to the worker and firm during the bargaining process. In one extreme case, firms are assumed to know not only the worker's current (or potential) productivity at their firm, but also the value of the worker's best alternative productivity match (e.g., Postel-Vinay and Robin (2002), Dey and Flinn (2005), Cahuc et al. (2006)). An alternative assumption is that employers either do not know the employee's outside option or that they simply don't respond to such information when making an offer.

In an early version of this paper, we showed that estimating a model of on-the-job search with surplus division and binding minimum wages yielded very different model estimates and policy implications depending on whether one assumed renegotiation or not.[2] Within their setting, which was more restrictive than the one considered here, in the no renegotiation case firms simply split the surplus with workers using the value of unemployed search as an outside option.[3] It is worth noting that both specifications of the bargaining process imply efficient mobility decisions, that is, a worker always left their current employer when she encountered a firm at which her productivity was greater. Facing the same primitive parameters characterizing the search environment, firms

[1]These assumptions are relaxed when taking the model to data, as is they imply a monotone increasing density function on the support of the wage distribution. For the estimation of such a model, see Bontemps et al. (2000))

[2]In these two cases, the primitive parameters characterizing the models are identical.

[3]Whlle this is clearly restrictive, the value of unemployed search is defensible as an outside option in the case that firms cannot commit honoring previous wage commitments after an applicant's or worker's competing wage offer has been withdrawn.

prefer a world in which there is no renegotiation, while individuals capture more of the surplus under renegotiation of contracts.

In this paper we consider a world in which there exists a positive measure of firms who renegotiate and a positive measure of firms that do not, with the proportion of firms of both types determined within an equilibrium model of vacancy posting. Firms that renegotiate have informational advantages with respect to those that don't, but also agree to a surplus division policy that commits them to increasing wages to workers even when the potential new job has lower productivity than the current one, but is greater than the employee's previous outside option under which the existing wage was set. Wage-posting firms issue wage contracts that are functions of the employee's productivity at the firm, and are fixed over time. Wage-posting firms are cognitive of the existence of renegotiating firms, and make conditional (on productivity) strategic wage offers that are functions of the measure of renegotiating firms in the labor market. In this environment, an employee of a non-renegotiating (n) firm who encounters a renegotiating (r) firm may leave the current employer even though their productivity is lower at the type r firm. In this world, there exists inefficient mobility, a phenomenon that does not exist if all firms are type n or if all firms are type r.

We assume that the vacancy posting costs are different for jobs at r or n type firms. One rationale for this assumption is that r type firms must invest in verifying an applicant's current outside option, including what their productivity level is if they are currently employed by another firm and what type of firm their current employer is (i.e., r or n). Since we assume that all firms are identical ex ante, we characterize the unique equilibrium in which firms are indifferent between posting an r or an n type vacancy, and the expected value of a vacancy of either type is 0. Denote the equilibrium proportion of type r firms by $p(\Omega)$, where Ω is the set of primitive parameters characterizing the model. One of our goals is to determine how the mixture of firm types is affected by changes in the environment. Our policy application will be to the impact of minimum wages on labor market outcomes. In this case, the economic environment is described by Ω and m, the minimum wage. The proportion of firms of type r under the minimum wage is given by $p(\Omega, m)$, and our interest is in determining the equilibrium effects of a minimum wage change through this channel. As noted above, workers' surplus is generally maximized when all firms are type r. While a minimum wage increase will typically reduce wage posting, thereby attenuating or reversing gains to workers in partial equilibrium, in this case the impact will further depend on how the minimum wage affects the proportion of type r firms in the economy.

The most similar paper to the current one is Postel-Vinay and Robin (2004). In that paper, the authors utilize a matching technology with workers of heterogeneous abilities, a, and firms of heterogeneous productivities, z, coming together through random search, with the productivity of the match given by az. They attempt to find a separating equilibrium, in which firms with productivities in the set R agree to negotiation while those with $z \notin R$ refuse. Under certain restrictions, they are able to prove existence of an equilibrium and perform some numerical experiments. Unlike their paper, ours assumes ex ante homogeneity of firms, which is partially dictated by the sources of data at our disposal. In our case, we assume that the productivity of an individual of type a is given by $a\theta$, where θ is an i.i.d. match draw representing the worker's idiosyncratic productivity associated with working at a given firm. Our model also features the endogenous determination of vacancy creation. One of the main focuses of our paper has how changes in the economic environment (e.g., a minimum wage) affects the mix of the two types of firms in the economy.

In Section 2 we describe the model and present some results. Section 3 introduces a minimum wage into the model. In Section 4 we discuss our choice of data that will inform an empirically plausible parameterization of the model, which we arrive at through an indirect inference procedure. Section 5 describes and presents the results of this procedure. The resulting estimates allow us to quantitatively explore some implications of the model in partial equilibrium. In Section 6 we extend the model to include the endogenous determination of contact rates, and we perform policy experiments in which the minimum wage is varied. Section 7 concludes.

2 Model

2.1 Setup and Preliminaries

The model is set in continuous time, with all agents on the supply side of the market distinguished by their ability, a. Upon meeting any firm in the market, individuals draw a productivity realization θ. a and θ are distributed on a subset of the positive real line with c.d.f. F_a and F_θ, respectively. It is necessary for us to assume that F_θ is continuous on it's domain, while we do not, in principal require any such restrction on F_a. The value θ, as well as ability a, are perfectly observed by both (potential) employees and firms, and productivity realizations are independently distributed across employee-employer pairs. An employee with ability a at a firm with match θ produces a flow output $a\theta$, while an unemployed worker enjoys flow utility ab.

Firms in the market are *ex ante* homogeneous except for the manner in which they interact with potential or current employees in setting wages, the only utility-yielding characteristic of the employment contract to the worker. The firm's bargaining type is indicated by j, with $j \in \{r, n\}$. A type r firm is a "(re-)negotiator," and this type of employer bargains over wage contracts with employees at the beginning and over the course of their tenure at the firm. A type n firm is a "nonrenegotiator," that makes a one-time take-it-or-leave-it wage offer to a potential employee based upon the individual's ability, a, and potential productivity at the firm, θ. In the remainder of this paper, we adopt the semantic convention of referring to a single firm as an "r-firm" or an "n-firm". The value to workers of being at either type of firm is summarized by the value function V_j, $j \in \{r, n\}$.

In the steady state, unemployed workers meet firms at rate λ_u, while workers encounter alternative employers at a rate λ_e. Matches are exogenously destroyed at a constant rate δ. When meeting a potential employer, the probability is p that it is of type $j = r$. In section 6 we show how the contact rates λ_u and λ_e, as well as the proportion, p, of r-type firms are determined in general equilibrium. A critical assumption that our solution requires is the *free entry condition*: the expected return to market entry (achieved by purchasing and posting a vacancy) for either type of firm is equal to zero.

In the remainder of this section we focus our attention on how to solve for several important endogenous objects in equilibrium. We proceed by:

1. Introducing the wage-bargaining framework for r-firms (Section 2.2).
2. Solving for the worker's value functions, V_n and V_r, and mobility decisions, holding fixed the wage-offer strategies of n-firms (Section 2.3).
3. Under the given rules for worker mobility, solving for the distribution of workers in steady state across employment states (Section 2.4).
4. Fixing the above endogenous objects, we solve the wage-offer problem faced by an n-firm. To close the model, n-firms' optimal wage offer strategies must be in concordance with those we fixed in step (2). We show how to solve the model under this equilibrium restriction (Section 2.5).
5. Finally, we consider the implications of adding a binding minimum wage to the model (Section 3).

A note on heterogeneity in the model To simplify exposition, we suppress dependence of the model's value functions and wages on ability, a. Since wages at both firm types can be conditioned on ability, the reader can think of the following model solution as applying for fixed a.

2.2 Wage-Setting At R-Firms

While n-type firms make non-negotiable and permanent offers, we must describe in more detail the set of assumptions that define wage determination for renegotiating firms in equilibrium. Importantly, wages are set such that the value afforded to workers is equal to their private outside option plus a share, α, of the *joint surplus* generated by the match. Let $S(\theta)$ denote the joint surplus available from a match θ, so that for example, when hiring a worker from an n-type firm, with a wage w, a wage is bargained that results in a continuation value to the worker of:

$$(1-\alpha)V_n(w) + \alpha S(\theta)$$

Similarly, when hiring the worker out of unemployment, the continuation value achieved is:

$$(1-\alpha)V_u + \alpha S(\theta)$$

When, during the bargaining process, the worker currently has a job at an r-type firm, we assume that both firms are drawn into Bertrand competition. In this setting, the outcome is identical to that in Cahuc et al. (2006) and Dey and Flinn (2005): the losing firm is willing to pay a wage up to, but not exceeding the value of the match, θ'. Thus, the worker's outside option in this case is the full surplus of the match $S(\theta')$ and hence she receives the continuation value:

$$(1-\alpha)S(\theta') + \alpha S(\theta).$$

2.3 Values and Match Surplus Equations

Before describing the full set of mobility patterns that can occur in equilibrium, it will be useful to write down and investigate the properties of the surplus function, S, and the worker's value function, V_n, at non-negotiating firms. To do this, let Φ be the endogenous distribution of offers received from non-negotiating firms (we will later turn our attention to solving for this object in equilibrium). Under this assumption,the value to a worker at an n-firm can be written as follows:

$$(r+\delta)V_n(w) = w + \underbrace{\lambda_e p \int \alpha[S(x) - V_n(w)]^+ dF_\theta(x)}_{(1)} + \underbrace{\lambda_e(1-p)\int [V_n(x) - V_n(w)]^+ d\Phi(x)}_{(2)} + \delta V_u. \quad (1)$$

Here, term (1) is the expected continuation value derived when the worker meets an r-firm, which occurs at a rate $\lambda_e p$. If the surplus attainable at this firm exceeds the value of remaining, a fraction α of the difference is obtained through the Nash-bargaining process. Term (2) is the expected continuation value derived when meeting another n-firm, which is seized upon only if the value from the offered wage exceeds the value of remaining.

The total surplus function, S, can be similarly written below. This object, which is the total value to both the worker and the firm from the match, is useful because in our framework we have assumed directly transferable utility. However, it may help the reader to imagine that $S(\theta)$ is the value to the worker when their wage is equal to total match output (and hence they have captured the full surplus from the match):

$$(\rho+\delta)S(\theta) = \theta + \underbrace{\lambda_e p \int \alpha[S(x)-S(\theta)]^+ dF_\theta(x)}_{(1)} \\ + \underbrace{\lambda_e(1-p)\int [V_n(x)-S(\theta)]^+ d\Phi(x)}_{(2)} + \delta V_u \quad (2)$$

Once again, term (1) shows what happens when the worker meets another r-firm: Bertrand competition bids up the worker's outside option to $S(\theta)$ and an additional fraction α of the difference is obtained through bargaining. The firm receives a value of 0, by virtue of the free entry condition. In term (2), the worker meets an n-firm and additional surplus is only generated if the value from the wage offer exceeds $S(\theta)$. In this case, the firm once again receives 0.

An immediate observation is that setting $V_n = S$ allows both recursive equations to hold (in fact they become identical), and hence we conclude that $S(x) = V_n(x)$. This allows us to write a dynamic program solely in terms of S:

$$(\rho+\delta)S(\theta) = \theta + \lambda_e p \int \alpha[S(x)-S(\theta)]^+ dF_\theta(x) + \lambda_e(1-p)\int [S(x)-S(\theta)]^+ d\Phi(x) + \delta V_u. \quad (3)$$

The substance of this useful result is that earning a wage w at an n-firm is, from the worker's perspective, equivalent to being at an r-firm with match productivity w, having claimed the full surplus from this match. It is elementary to now show that this surplus function is *strictly increasing* in its sole argument. This permits us to define the *reservation match value*, θ^*, according to:

$$V_u = S(\theta^*) \quad (4)$$

This concept defines which matches (and wage offers from n-firms) are acceptable to workers when being hired out of unemployment.

Additionally, this observation proposes a useful state definition for workers at both type of firms: the *maximum attainable wage*, x. At r-firms, the maximum attainable wage is equal to the match productivity, θ, while at n-firms, it is equal to w (since the firm is unwilling to renegotiate w). This state, x, is particularly useful for parsimoniously writing the worker's value function when employed at an r-firm, and for describing mobility patterns and renegotiation in equilibrium.

To verify the second point, note that since S is a monotonic function and equal to V_n, the worker will move to the firm at which the maximum attainable wage is highest, since this criterion identifies the firm that is able to offer the highest value to the worker after the option to renegotiate is exercised.

To address the first point, notice that we can now write the value to a worker at an r firm as:

$$V_r(\theta, q) = (1-\alpha)S(q) + \alpha S(\theta). \quad (5)$$

Here, q is defined as the maximum attainable wage at the outside option used when the current wage was bargained. According to our assumptions, this is equal either to the wage offer at n-firms or the match productivity at r-firms. The state variable q also parsimoniously defines when a wage must be renegotiated: when the negotiated wage, using this outside option, achieves a value greater than the one currently bargained. Inspection of (5) reveals that this can only occur when a maximum attainable wage, x, is drawn such that $x \geq q$. With this observation in hand, we can now write the recursive definition of this value function:

$$
\begin{aligned}
&(\rho + \delta + \lambda_e p \overline{F}_\theta(q) + \lambda_e(1-p)\overline{\Phi}(q))V_r(\theta, q) = \phi(\theta, q) \\
&\qquad + \lambda_e p \left[\underbrace{\int_q^\theta [(1-\alpha)S(x) + \alpha S(\theta)] dF_\theta(x)}_{(1)} + \underbrace{\int_\theta [\alpha S(x) + (1-\alpha)S(\theta)] dF_\theta(x)}_{(2)} \right] \\
&\qquad\qquad + \lambda_e(1-p) \left[\underbrace{\int_q^\theta [(1-\alpha)S(x) + \alpha S(\theta)] d\Phi(x)}_{(3)} + \underbrace{\int_\theta S(x) d\Phi(x)}_{(4)} \right] + \delta V_u \qquad (6)
\end{aligned}
$$

Here the wage $\phi(\theta, q)$ is set such that the surplus split defined in (5) is achieved. The gains in dynamic value arise from four different outcomes, described as follows: (1) The worker meets an r-firm and a match x is drawn that beats the previous outside option, q, and hence the wage at the incumbent firm is *renegotiated* after Bertrand competition; (2) The worker meets an r-firm and a match x is drawn that beats the current match θ. The incumbent firm competes for the worker, but is unwilling to bid a wage above θ, and hence the surplus $S(\theta)$ is used as an outside option when bargaining with the new firm; (3) The worker meets an n-firm and draws a non-negotiable wage offer x that beats the previous outside option, q, and the wage is renegotiated at the incumbent firm; and (4) The worker meets an n-firm and draws a non-negotiable wage offer x that beats the best available offer from the incumbent firm, θ. The wage function $\phi(\theta, q)$ can be derived by combining (6) with (5). We relegate its formal expression to the appendix.

To close this section we show how the value to workers from being unemployed, V_u, can be written. First, note that when hiring a worker out of unemployment, this is equivalent to hiring a worker from a firm with match productivity θ^*, and hence the worker's value can be written as $V_r(\theta, \theta^*)$ in this case. We get:

$$\rho V_u = b + \lambda_u p \alpha \int_{\theta^*} (S(x) - V_u) dF_\theta(x) + \lambda_u(1-p) \int_{\theta^*} (S(x) - V_u) d\Phi(x). \qquad (7)$$

Using (4) with the above we can define the reservation match quality θ^* by the relation:

$$\theta^* = b + (\lambda_u - \lambda_e) \left[p\alpha \int_{\theta^*} (S(x) - S(\theta^\star)) dF(x) + (1-p) \int_{\theta^*} (S(x) - S(\theta^\star)) d\Phi(x) \right] \qquad (8)$$

With the definition of θ^* now in hand, the following lemma will prove useful in the next section, so we introduce it here.

Lemma 1. *Define* $\underline{w} = \inf\{x : \Phi(x) > 0\}$. *Then* $\underline{w} = \theta^*$.

Proof. See appendix. □

This result follows immediately by noting that all matches $\theta > \theta^*$ are profitable for n-firms, while wage offers above the match value are not profitable.

2.4 Steady State

In the previous section, we derived a characterization of the conditions under and the rate at which workers move between employment states. In this section, we use those rules to derive the steady state distribution of workers across states.

First, normalizing the mass of workers in the economy to 1, we let M_e and M_u indicate the steady state mass of workers in employment and unemployment, respectively. From the previous section, we conclude that the flow rate out of unemployment is $\lambda_u \overline{F}_\theta(\theta^*)$, with $\overline{F}_\theta = 1 - F_\theta$ (we adopt this notational convention, $\overline{\cdot}$ for all distributions). The flow rate into unemployment is simply δ. Thus, we can write:

$$M_e = \frac{\lambda_u \overline{F}_\theta(\theta^*)}{\delta + \lambda_u \overline{F}_\theta(\theta^*)}, \qquad M_u = \frac{\delta}{\delta + \lambda_u \overline{F}_\theta(\theta^*)}.$$

We established in the last section that worker mobility is defined by a sufficient statistic: the *maximum attainable wage*. Let G be the distribution of workers across this state. G represents the distribution of best available offers, conditional on being employed. We can see how this object is required knowledge when an n-firm makes its wage offer, since it must factor the probability that a given wage is acceptable to the prospective employee. For a randomly sampled employed worker, the probability that a wage, w, is acceptable is given by $G(w)$.

Let $G_r(x)$ and $G_n(x)$ be the measure of workers at n and r firms with best attainable wage less than or equal to x. We know that:

$$G_r(x) + G_n(x) = G(x)$$

Finally, let $H(\cdot \mid x)$ be the conditional distribution of "most recent competing offers" for workers at r firms with match x. For example, $H(q \mid x)$ is the probability that a worker at a firm with match x used an outside offer less than or equal to q for their most recent wage-bargain.

Balancing flow equations in Appendix D.1 gives the following closed-form expressions for each distribution object:

$$G(x) = \frac{p(F_\theta(x) - F_\theta(\theta^*)) + (1-p)(\Phi(x) - \Phi(\theta^\star))}{\delta + \lambda_e p\overline{F}_\theta(x) + \lambda_e(1-p)\overline{\Phi}(x)} \frac{M_u}{M_e} \tag{9}$$

$$g_r(x) = \frac{\lambda_u p f_\theta(x)(\delta + \lambda_e \overline{F}_\theta(\theta^*)}{\Psi(x)^2} \frac{M_u}{M_e} \tag{10}$$

$$g_n(x) = \frac{\lambda_u(1-p)\phi(x)(\delta + \lambda_e \overline{F}_\theta(\theta^*)}{\Psi(x)^2} \frac{M_u}{M_e} \tag{11}$$

$$H(q|x) = \left(\frac{\Psi(x)}{\Psi(q)}\right)^2 \tag{12}$$

where

$$\Psi(x) = \delta + \lambda_e p\overline{F}(x) + \lambda_e(1-p)\overline{\Phi}(x)$$

This final term, $\Psi(x)$, defines the flow rate of exit of a worker from a firm whose highest possible wage offer is x.

With these expressions we have fully characterized, given the endogenous wage offer distribution Φ, how workers are distributed across firms and wages in steady state. We can finish presenting the model solution then by showing how the distribution Φ can be derived, and imposing that it is consistent with the equilibrium conditions presented thus far.

2.5 The Firm's Problem

So far we have established, given our bargaining and renegotiation assumptions, how the value functions for workers and for r-firms can be determined, fixing the distribution Φ of wage offers from n-firms. To round out the model, we introduce the wage-setting problem of n-firms and show how it can be solved. We assume that, upon meeting a worker, a match quality θ is drawn. Next, the n-firm makes a *non-negotiable* wage offer, w, that the worker can accept or reject. If the wage is rejected, the match is dissolved instantly. The worker takes her outside option in this case, while the firm receives zero payoff, by virtue of the free entry condition. We assume that the firm must make its offer with no information about the worker's current employment state. This assumption may seem stark, since one could imagine that a firm can acquire such information with ease. However, we view pursuit of this strategy as being tantamount to the firm "coming to the table" in the parlance of bargaining, and hence the "take-it-or-leave-it" nature of the offer no longer becomes credible.

Thus, firms' expected discounted profit can be written as:

$$J(\theta, w) = \underbrace{\text{HireProb(w)}}_{(1)} \times \underbrace{\frac{\theta - w}{\rho + \delta + \lambda_1 p\overline{F}_\theta(w) + \lambda_1(1-p)\overline{\Phi}(w)}}_{(2)}.$$

Term (1) is simply the probability that the wage offer w is accepted by the worker, while term (2) is the discounted value of profits. The numerator of this term is the firm's flow profit, while the denominator reflects the effective discount rate, which incorporates the hazard rate at which the firm loses the worker. This occurs whenever the worker meets an r-firm and a match is drawn that beats w, or whenever the worker meets an n-firm and a wage is drawn that beats w.

Restricting our attention to wage offers that satisfy $w \geq \theta^*$ (since wage offers less than θ^* are trivially suboptimal), the hiring probability can be written as:

$$\text{HireProb}(w) = \underbrace{\frac{\lambda_u M_u}{\lambda_u M_u + \lambda_e M_e}}_{(1)} + \underbrace{\frac{\lambda_e M_e}{\lambda_u M_u + \lambda_e M_e}}_{(2)} G(w).$$

With probability given by term (1), the worker is unemployed, and the match is accepted. With probability given by term (2), the worker is employed. In this case, the offer is only accepted if the worker's current maximum attainable wage is less than w. This happens with probability given by $G(w)$.

Using our derivation of G from the previous section, we can write the firm's expected profit finally as

$$J(\theta, w) = \frac{\lambda_u M_u \lambda_e \overline{F}_\theta(\theta^*)}{\lambda_u M_u + \lambda_e M_e} \frac{\theta - w}{\Psi(w)(\rho + \Psi(w))} = \Gamma(w)(\theta - w) \tag{13}$$

Subject to a match draw θ, the firm solves the problem

$$\max_w J(\theta, w). \tag{14}$$

Notice that if this problem identifies a unique wage, w, for each match level θ, then this defines a function $\varphi : \mathbb{R}^+ \mapsto \mathbb{R}^+$. If this function, φ, is strictly increasing, then we can write:

$$\Phi(w) = F_\theta(\varphi^{-1}(w)).$$

To clarify the role played by Φ as a functional parameter of the maximization problem, let us re-write (14) as:

$$\max_w \{\Gamma(\Phi(w), F_\theta(w))(\theta - w)\} \tag{15}$$

Given a monotonically increasing offer function, φ, this defines an operator:

$$[\mathcal{T}\varphi](\theta) = \arg\max_w \left\{\Gamma(F_\theta(\varphi^{-1}(w)), F_\theta(w))(\theta - w)\right\}.$$

In equilibrium, it must be that the wage offer function φ is a fixed point of this operator $\mathcal{T}$, i.e. $\mathcal{T}\varphi = \varphi$. Before we show how such a fixed point can be found, the following result guarantees that searching for such a deterministic, monotonic function is appropriate in this setting.

Proposition 1. *Firms' optimal wage offer strategies are given by a deterministic function φ that is (1) monotonically increasing; (2) lower semi-continuous; (3) almost everywhere differentiable; and (4) satisfies $\varphi(\theta^*) = \theta^*$.*

Proof. The proof is given as a combination of Lemmas 1-7 in the Appendix. $\square$

We finish this section by proposing a parsimonious computational strategy for finding the fixed point, φ. Notice that, if such a solution is found, the model in partial equilibrium (i.e. with fixed values of p, λ_u, and λ_e) is solved. Inspecting equation (13) reveals the trade-off that firms face: both hiring and worker retention probabilities are *increasing* in w, while flow profits are decreasing in w. These are, as usual, balanced by the first-order condition:

$$\frac{d}{dw}\Gamma\big(F_\theta(\varphi^{-1}(w)), F_\theta(w)\big)(\theta - w) - \Gamma(F_\theta(\varphi^{-1}(w), F_\theta(w)) = 0$$

Using Γ_1, Γ_2, to denote the derivative of Γ in its first and second arguments, we get:

$$\Big[\Gamma_1\big(F_\theta(\varphi^{-1}(w)), F_\theta(w)\big)f_\theta(\varphi^{-1}(w))/\varphi'(\varphi^{-1}(w)) \\ + \Gamma_2\big(F_\theta(\varphi^{-1}(w)), F_\theta(w)\big)f_\theta(w)\Big](\theta - w) - \Gamma\big(F_\theta(\varphi^{-1}(w)), F_\theta(w)\big) = 0 \quad (16)$$

Finally, by imposing that in equilibrium we must have $\varphi(\theta) = w$, this condition becomes

$$\Big[\Gamma_1\big(F_\theta(\theta), F(w)\big)f_\theta(\theta)/\varphi'(\theta) + \Gamma_2\big(F_\theta(\theta), F(w)\big)f(w)\Big](\theta - w) - \Gamma\big(F_\theta(\theta), F_\theta(w)\big) = 0 \quad (17)$$

This can be rearranged into a first order differential equation which, when combined with the boundary condition $\varphi(\theta^*) = \theta^*$, is readily solved numerically. Notice that Proposition 1 does not guarantee that the first order condition uniquely identifies the optimal wage offer, and in fact there may be discontinuities in φ. In Appendix D.2 we show how to leverage the properties of the wage solution into a robust numerical algorithm.

2.6 Introducing Heterogeneous Ability

In delivering the above model solution, we asked the reader to relax dependance of the model's endogenous objects on a. Before we move to the next section, we note that this dependance can be re-introduced by verifying that the endogneous system of objects: $S, V_n, \phi, \varphi, V_u$ are all multiplicative in a. Thus, we can write:

$$\begin{aligned} S(a,\theta) &= aS(\theta) \\ V_r(a,\theta,q) &= aV_r(\theta,q) \\ \phi(a,\theta,q) &= a\phi(\theta,q) \\ \varphi(a,\theta) &= a\varphi(\theta) \\ V_u(a) &= aV_u \end{aligned}$$

The above properties also imply that the reservation match value θ^* is invariant in a. This simplification is guaranteed by the fact that (1) the flow value of unemployment (ab) and output ($a\theta$) are multiplicative in a; and (2) firms can condition their wage offers on a. We can, therefore, think of $\phi(\theta, q)$ and $\varphi(\theta)$ as efficiency wages, a fact we exploit in estimating this version of the model. In the next section we will see that this convenient property is lost once the possibility of a binding minimum wage is introduced.

3 Introducing a Minimum Wage

In this section we extend the model to allow for a binding minimum wage, m. There are two consequences of the minimum wage for wage-setting in this model. First, supposing that $a\theta^* < m$, the minimum wage renders matches in the range $[\theta^*, m/a]$ unacceptable. Second, even if the reservation match value θ^* satisfies $a\theta^* > m$, the minimum wage may still bind. This will occur at firms where the continuation value is large enough such that a worker is willing to accept a wage less than the legal minimum: $a\phi(x, y) < m$. In order to think about the problem for a worker of ability level a, we recast the problem in terms of efficiency wages (as we did in the last section). Accordingly, we define the efficiency minimum wage $\tilde{m} = m/a$, which tells us the minimum allowable efficiency wage paid to worker a. In what follows, we fix this ability level, a, and for notational simplicity suppress dependence of functions on this parameter. Additionally, note that there will be ability levels sufficiently high such that the minimum wage does not bind in any case and is therefore irrelevant to the model.

Now the model can once again be thought of as multiplicatively homogenous in a. Let $M(\theta)$ be the value to a worker of earning the minimum wage, $\tilde{m}$, at a firm with match θ. Noting that values are monotonically increasing in the wage earned, we note that the minimum wage will be paid whenever $\phi(\theta, y) < \tilde{m}$, which occurs whenever

$$\alpha V(\theta) + (1 - \alpha)V(y) < M(\theta),$$

Thus, the value function M can be written recursively as:

$$\begin{aligned}(\rho + \delta)M(\theta) = \tilde{m} &+ \lambda_e p \int_{\tilde{m}}^{\theta} \max\{(1 - \alpha)S(x) + \alpha S(\theta) - M(\theta), 0\} dF_\theta \\ &+ \lambda_e p \int_{\theta} \max\{M(x) - M(\theta), \alpha S(x) + (1 - \alpha)S(\theta) - M(\theta)\} dF_\theta \\ &+ \lambda_e (1 - p) \int \max\{(1 - \alpha)S(x) + \alpha S(\theta) - M(\theta), S(x) - M(\theta), 0\} d\Phi + \delta V_u \qquad (18)\end{aligned}$$

When a worker, earning minimum wage at a firm with match θ, meets another firm with best available wage x, the outcomes can be described as follows:

1. With probability p, the firm is willing to negotiate, and if $\tilde{m} < x < \theta$, our usual bargaining assumptions apply, with the worker negotiating a new wage offer $\phi(\theta, x)$. However, it may be that the negotiated wage does not beat $\tilde{m}$ in which case the generated surplus is zero.

2. If the competing firm is willing to negotiate and $x > \theta$, then the wage offer $\phi(x, \theta)$ beats

the best available offer from the incumbent, and the worker moves. However, it may still be that $\phi(x, \theta) < \tilde{m}$, in which case the worker may claim the minimum wage.

3. With probability $(1 - p)$, the firm makes a non-negotiable offer, x. In this case, the worker may accept a newly bargained wage at the incumbent firm, $\phi(\theta, x)$, she may accept the non-negotiable wage x and switch firms, or she may prefer to keep her minimum wage offer at the incumbent firm.

The three outcomes here correspond to the three integrals on each line of the above expression.

Accordingly, the new surplus function S must be re-written to incorporate the possibility that future wages are constrained by m:

$$(\rho + \delta)S(\theta) = \theta + \lambda_e p \int_\theta \max\{M(x) - S(\theta), \alpha[S(x) - S(\theta)]\} dF_\theta \\ + \lambda_e(1 - p) \int_\theta (S(x) - S(\theta)) d\Phi + \delta V_u \quad (19)$$

We can, as before, define the reservation match value by the relation $S(\theta^*) = V_u$. Since there is no guarantee that $\theta^* \geq \tilde{m}$, we must define in addition $\theta^\star = \max\{\theta^*, \tilde{m}\}$ which gives the lowest profitable match. In turn, this defines the lower bound of the offer distribution for n-firms. Adding the restriction below on reservation matches to those above is sufficient to pin down the solution:

$$\rho S(\theta^*) = b + \lambda_e p \int_{\theta^\star} \max\{M(\theta), \alpha(S(\theta) - S(\theta^*)), 0\} dF_\theta + \lambda_e(1 - p) \int_{\theta^\star} (S(\theta) - S(\theta^*)) d\Phi \quad (20)$$

3.1 When Minimum Wages Bind

We next revisit the question of when minimum wages bind in this model. First, it must be noted that the conditions of Lemma 1 still apply here, and hence minimum wages never bind at n-firms. Figure 2 shows two scenarios in which the minimum wage will bind at r-firms. For illustrative purposes, we set the minimum wage at \$5.50 and hour. First, we fix the winning firm's match at \$6/hr, and consider what happens to $\phi(6, y)$ as y decreases. We can see in the left panel of Figure 2 that the bargained wage hits the lower bound of \$5.50 for lower values of y. In addition, we see that the presence of a binding minimum wage inflates *all* wages above what they would be in the non-binding case. In the right panel of Figure 2, we see what happens when the outside option is fixed at \$6/hr and the winning match y increases: for higher values of y, the minimum wage binds. As y increases, the worker is in principle willing to accept wage cuts in exchange for the possibility that they will capture future surplus, but the lower bound on wages prohibits such a tradeoff, transferring surplus to the worker.

Furthermore, these two scenarios trace out a combination of match value pairs (x, y) at which the minimum wage interferes with the bargaining process in this way. Specifically, when either $\phi(x, y) < m$ or $\phi(y, x) < m$, the bargaining process is rendered unnecessary by the minimum wage regulation. In Figure 3 we trace out this area for a particular parameterization, and call the resulting area the "No Bargain Zone".

4 Data

While Hall and Krueger (2012) provide compelling evidence that wages are set both through bargaining and non-negotiable offers, the data they collect is not informative about the other structural parameters of our model. In pursuit of estimation, we elect to use the *Survey of Income and Program Participation* (SIPP) which has been used successfully in the past to estimate models of OTJ search (Dey and Flinn, 2005; Flinn and Mullins, 2015). The SIPP is a nationally representative, household-based survey comprised of longitudinal panels. Each panel lasts four years in total. The survey is administered in four-month waves, at which point information is collected retrospectively for the previous four months. Therefore, each panel contains 12 waves of the survey. Our data is constructed from waves 3 through 8, [4] giving data on employment status and wages for a 24 month window, from 2004 to 2006. [5]

Since our principal application of the model developed in the previous section will be to the introduction of a binding minimum wage, we focus our attention on a subpopulation most likely to be affected by this change: high school graduates between the age of 21 and 30. Using the 2004 Panel of the SIPP, we construct a data set consisting of employment and unemployment spells for workers who fit this sample selection criteria.

In Appendix E we offer more precise details on how these data are constructed. Here we give sufficient detail for the reader to understand the analysis that follows. Our operating dataset is a panel D_i for each worker i of the form:

$$D_i = \{(E_{i,s}, W^m_{i,s,0}, W^m_{i,s,1}, T_{i,s}),\ s = 1,2,...,S_i\} = \{D_{i,s},\ s = 1,2,...,S_i\}$$

where $E_{i,s} \in \{0,1\}$ indicates the employment status of the worker in that spell, $T_{i,s}$ is the duration (in months) of the spell, while S_i is the number of spells observed for worker i. We use $D_{i,s}$ to denote the relevant data for spell s of worker i.

If employed ($E_{i,s} = 1$) then $W^m_{i,s,0}$ is the wage measured[6] at the beginning of the spell, and $W^m_{i,s,1}$ the wage at the end of the spell. If unemployed ($E_{i,s} = 0$), wage entries take null values. Spells at the beginning and end of the 24-month period are truncated, in which case we record the truncated durations, letting $W^m_{i,1,0}$ and $W^m_{i,S_i,1}$ be the measured wage at the beginning and end

[4] See Appendix E for the reasons that motivate our choice of these waves

[5] Because of the SIPP's rotating wave structure, the beginning and ending months of each wave are not identical for all survey members

[6] This notation allows us to assume later that the true wage $W_{i,s,j}$ and the measured wage in the data are separated by measurement error.

of the sample window, respectively. Since our key solution concept is the notion of *steady state equilbrium*, we must assume that the economy is in steady state when we draw our sample. Under this assumption, employment status and wages in the first observed spell, $\{E_{i,0}, W^m_{i,0}\}$, which is taken at the beginning of the observation window, can be thought of as a random draw from this steady state. This observed spell can be used, therefore, when thinking about the distribution of wages and employment states in the stationary equilibrium.

In Table 2 we present some descriptive statistics from this data set. Several features of the data are of note. To begin, notice that workers in this sample face long unemployment durations, 5.847 months on average. Accordingly, the steady state rate of "unemployment" is high, around 20%. There are two explanations for this statistic. First, a higher unemployment rate for this selected sample of low-skill workers is demographically consistent with previous work. Second, we are classifying "unemployment" somewhat differently from traditional studies of labor-market flows, where sample members must report actively looking for work to be counted among the unemployed. Rather, we designate unemployment to be any observed absence of an employment spell. Given the documented pattern of movement between traditional definitions of unemployment and the designation of being "out of the labor force"(Flinn and Heckman, 1983), our inflated unemployment rate is not incoherent with the literature.

Despite the relative slackness of this labor market, we find that OTJ transitions are not uncommon. On the contrary, we document that 31.8% of the employment spells that end (i.e. that are not truncated by the 24-month cut-off) in our sample are ended by a transition to a new employer.

We also make note of the average wage for workers in this sample. At roughly \$14 an hour in 2016 dollars,[7] this implies that many of our sample workers would be affected by recent minimum wage proposals, which have ranged between \$10 and \$15 an hour. Conversely, a \$15 minimum wage in 2016 dollars corresponds to a minimum wage of roughly \$11.80 in our sample. By inspecting Figure 1, which shows the distribution of workers' wages, we can see that a sizeable fraction of the sample would indeed be affected by such a change.

To finish this section, we revisit the statistics provided by the survey of Hall and Krueger (2012), which are presented in Table 1. We separate workers by those who have obtained a high school diploma or less, and those who have attended some college. The pattern of increasing bargaining rates for higher-skilled workers is clearly observable, with college-attending workers doing so at a rate of 40.8% compares to high school workers, 22.7% of whom report bargaining. In order to make appropriate comparison with our selected SIPP sample, we also examine the 74 workers in this survey who report high school attendance and have ages between 21 and 30.

[7] A back of the envelope calculation using the Bureau of Labor Statistics' Consumer Price Index (CPI) series. See https://download.bls.gov/pub/time.series/cu/

While this adds considerable imprecision to the estimate of the bargaining proportion, we feel that 15.5% is a reasonable fraction for this population, and uncertainty around this number can easily be accommodated in our estimation procedure. Finally, while college workers enjoy the highest premium to bargained wages (nearly $7 an hour), a small, yet significant, premium can be detected amongst the full sample of high school educated workers. The existence of such a premium is a prediction that can be revisited in our estimated model.

5 Estimation

Using the panel dataset constructed from the SIPP in the previous section, we proceed now to the problem of estimating the structural parameters of the model. We pursue this by judicious choice of statistics from the data, $\mathcal{S}_N$, for which we can construct model-based equivalents, $\mathcal{S}(\Omega)$, from simulated data, for a given choice of parameters, Ω. If $\mathbf{W}$ is a symmetric, positive-definite weighting matrix, then the estimator

$$\hat{\Omega}_N = \arg\min(\mathcal{S}_N - \mathcal{S}(\Omega))'\mathbf{W}(\mathcal{S}_N - \mathcal{S}(\Omega))$$

can be thought of as a simulated, minimum distance estimator. Commonly known as *Indirect Inference*, regularity conditions on the asympotic properties of $\mathcal{S}_N$ as sample size $N \to \infty$ guarantee that the estimator $\hat{\Omega}_N$ is itself consistent and uniformly asymptotically normal (Gourieroux et al., 1993).

Let us begin by considering which features of the data can be used for this purpose. Estimation of the rate parameters $\{\lambda_u, \lambda_e, \delta\}$ is straightforward using employment and duration data, following the analysis of Flinn and Heckman (1982) and Postel-Vinay and Robin (2002). We use the mean duration of employment and unemployment spells which (fixing other parameters) are tightly linked to the contact rate in unemployment, λ_u, and the total arrival rate of potentially spell-ending events, $\lambda_e + \delta$. We target as a third moment the fraction of completed employment spells that result in a job-to-job transition.

For the remaining parameters, we consider the distribution of log-wage changes under a set of employment histories. Noting that, in our model, $W_{i,s,j} = a_i\omega_{i,s,j}$ where $\omega_{i,s,j}$ is either equal to $\phi(\theta_{i,s}, q_{i,s,j})$, or $\varphi(\theta_{i,s})$, depending on the firm that currently employs worker i, we see that:

$$\log(W_{i,s,j}) - \log(W_{i,t,k}) = \omega_{i,s,j} - \omega_{i,t,k} \qquad \forall i, s, t, j, k.$$

Thus, examining log-wage differences neutralizes the role played by ability and allows us to separately estimate the remaining structural parameters without considering the distribution of ability. Furthermore, we wish to account for the possibility of measurement error in wages, given that wages are self-reported in the SIPP. Thus, we assume the following relationship between wages in the data and the true wage:

$$W_{i,s,j}^m = \epsilon_{i,s,j} W_{i,s,j}$$

where the measurement error, $\epsilon_{i,s,j}$, is drawn independently and identically over time from the distribution F_ϵ, with the restriction that $\mathbb{E}[\epsilon_{i,s,j}] = 1$. Assuming multiplicative measurement error, we get:

$$\log(W_{i,s,j}^m) - \log(W_{i,t,k}^m) = \omega_{i,s,j} + \log(\epsilon_{i,s,j}) - \omega_{i,t,k} - \log(\epsilon_{i,t,k}) \qquad \forall i, s, t, j, k.$$

Following this discussion, then, there are three distributions to estimate: F_a, F_θ, and F_ϵ. Let $h = \{d_1, d_2, ..., d_K\}$ be a particular employment history (a sequence of employment transitions). Our pursuit of identification relies on the fact that, given the finite structural parameters of the model ($\Omega = \{\lambda_u, \lambda_e, \delta, \alpha, p, b\}$), conditioning on a particular employment history, h, provides a log-wage change distribution defined by a parametric operator on the match distribution:

$$F_{\Delta \log(W) \ | \ H_i = h} = \mathcal{O}_{h,\Omega}[F_\theta]$$

Let us denote the corresponding characteristic function of $F_{\Delta \log(w)|h}$ as $\varphi_{\Delta\omega|h}$. Once our observations are overlayed with measurement error, we have the characteristic function decomposition:

$$\varphi_{\Delta \log(W^m)|h}(t) = \varphi_{\Delta\epsilon}(t)\varphi_{\Delta\omega|h}(t)$$

Thus we estimate the model by attempting to match the distribution of wage changes under three different employment histories:

$$\log(W_{i,s+1,0}^m) - \log(W_{i,s,1}^m) \mid E_{i,s} = 1,\ E_{i,s+1} = 1 \tag{EE}$$

$$\log(W_{i,s+2,0}^m) - \log(W_{i,s,1}^m) \mid E_{i,s} = 1,\ E_{i,s+1} = 0,\ E_{i,s+2} = 1 \tag{EUE}$$

$$\log(W_{i,s,1}^m) - \log(W_{i,s,0}^m) \mid E_{i,s} = 1,\ T_{i,s} = 24, W_{i,s,0}^m \neq W_{i,s,1}^m \tag{EE24}$$

The last line reflects the length of the panel we use, which is 24 months. Identification now rests on these three histories being sufficiently informative to invert out F_θ and $F_{\Delta \log(\epsilon)}$. Assuming that $F_{\log(\epsilon)}$ is symmetric, the latter is sufficient to identify it up to a location normalization, which we have made already. In the absence of measurement error, it is conceivable that even using only one of these histories would be sufficient, equivalent to the operator $\mathcal{O}_{h,\Omega}$ being invertible for history h. Since each distribution is convoluted with $\Delta \log(\epsilon)$ however, we add two more histories to ensure there are adequate restrictions on the model.

Typically, unless distributions can be directly analytically inverted from the data [8], nonparametric identification under these conditions requires assumptions of invertibility on the operator defined by the model. It is typically quite burdensome to prove such properties, so we do not

[8] Postel-Vinay and Robin (2002) is a relevant example of such a case

undertake that exercise in this paper. We will, however, find that estimation permits a relatively flexible specification of the distributions F_θ and F_ϵ. Specifically, we allow each to be a mixture of two log-normal distributions. F_θ therefore has parameters $\mu_1, \mu_2, \sigma_1, \sigma_2$ with mixing parameter π_θ, and F_ϵ an equivalent set. We normalize $\mu_1 = \mu_2 = 0$, while we choose $\mu_{\epsilon,1}, \mu_{\epsilon,2}$ such that $\mathbb{E}[\epsilon_{i,s,j}] = 1$.

Finally, to estimate the distribution of ability, F_a, we note that this can be nonparametrically estimated from the distribution of wages in steady state, assuming we have in hand estimates of all the other parameters. This follows from the observation that

$$\varphi_{\log(W^m)}(t) = \varphi_a(t)\varphi_{\log(\epsilon)}(t)\varphi_\omega(t)$$

Where the latter two characteristic functions come directly from the estimates of the model parameters. In practice, however, since we will later be required to solve the model with binding minimum wages, in which the role played by ability is no longer neutralized, we choose a parsimonious 5-point distribution[9] for a.

5.1 Estimates

With this data, we estimate the model by indirect inference, matching the transition moments and deciles of the distributions described above. Estimates from this procedure are presented in Table 3. Our estimate of the flow rate out of unemployment, 0.115 , is lower than typical estimates in this literature, reflecting the longer spells of unemployment faced by workers in our sample. The flow rate of offers while on the job, 0.026 , implies a job offer arrival rate of roughly once every 30 months. While this may seem low, we note that it is larger still than the rate of exogenous separation from firms. Again, these magnitudes are necessary to fit the average duration of employment spells in our chosen sample.

Turning to model fit, Tables 4 and 5 show that the model does a good job of matching the features of the data that we deem to be important and structurally informative. Of additional interest, however, are the predictions provided by the model on objects not directly observed in the data.

Figure 4 shows the densities of the distribution of matches, F_θ, and the distribution of measurement error, F_ϵ. While these two random variables appear to show a comparable spread, our estimates suggest that wages, while often accurately measured, show the capacity for severe mismeasurement. Additionally, the estimates suggest that variation in match quality is non-trivial, and it is not uncommon for the quality of the match to diminish or enhance output by values as large as 20%.

[9] We find that 5 types is sufficient to accurately fit the deciles of the steady state wage distribution

We estimate that the structural proportion of bargaining firms, p, is 0.074, about half the proportion of bargained wages observed in steady state, 15.52. This can be rationalized by the fact that r-firms are able to retain workers more effectively than n-firms.

5.1.1 Wage offers at n-firms

In equilibrium, how do n-firms adjust their wage offers with match quality? In Figure 5, we plot wage offers as a function of match quality, $\varphi(\theta)$. Two observations are immediate. First, for low matches close to the reservation, θ^*, the wage offer function is flat. Accordingly, for lower values of matches, n-firms are able to claim a greater fraction of the total match surplus.

Second, the wage offer gets steeper as the match density increases. Higher match densities reflect a greater probability that winning offers close to the current match are drawn, which requires the equilibrium offer to be steeper in this region. Conversely, for higher match values, where the probability of a better match being drawn is low, φ is once again flat. This implies that n-firms can extract a greater fraction of the surplus for higher match values also. We can, in fact, compute an "implied" bargaining share to the workers when an n-firm is met, given by:

$$\alpha_n(\theta) = \frac{S(\varphi(\theta)) - V_u}{J_n(\theta, \varphi(\theta)) + S(\varphi(\theta)) - V_u} \tag{21}$$

The term in the numerator gives the surplus of the match to the worker, while the denominator gives the total surplus of the match to both parties. Figure 6 shows $\alpha_n(\theta)$ using the model solution implied by our estimates. At the reservation match, since the only acceptable offer is θ^*, the worker claims all the surplus in this case. However, since $\varphi(\theta)$ is flat in this region, the implied share to workers quickly drops, and increases as competition to retain the match intensifies. We see a decrease once more for higher matches, when the local density of F_θ decreases. For reference, we plot also in Figure 6 the estimate bargaining parameter to workers ar r-firms, α, which takes the value 0.192, yielding that the implied bargaining position of n-firms is at times better and at times worse than r-firms.

In Appendix D.2 we show that these two features of the wage function (flatness in the region close to θ^*, and flatness in regions of low density) are consistent across all parameterizations of the model.

5.1.2 Wage Inequality

A recurrent question in the empirical literature on search frictions concerns the extent to which search frictions can account for observed inequality in worker's wages (Postel-Vinay and Robin, 2002; Hornstein et al., 2011). Our extension of standard OTJ search models to include both negotiating and non-negotiating firms has unique implications for this question. To facilitate this analysis, we repeat the observation that wages can be decomposed as:

$$\log(W_i) = \log(a_i) + \log(\omega_i)$$

where ω_i is equal to $\phi(\theta_i, q_i)$ if the worker is at an r-firm, and $\varphi(\theta_i)$ if at an n-firm. Thus, overall wage inequality can be conveniently decomposed into a component derived from ability, a_i, and a component derived from search frictions, ω_i. In Figure 7 we plot the distribution of the component ω, which we will call "residual wages" in steady state at n and r-firms. We see that, while workers at both types of firms display dispersion in this wage residual, the distribution of wages at r-firms shows much more residual wage inequality. Equivalently, we can say that the existence of n-firms, who do not renegotiate wages, compresses the dispersion in wages attributable to search frictions. The logic behind this result is simple. For lower match values, while n-firms are forced to offer wages at or near the reservation match θ^*, r-firms can offer much lower wages, which the worker is willing to accept under the knowledge that wage increases are likely in the future. At higher match values, workers at r-firms are able to obtain consistently higher wages (and greater fractions of the match) through encounters on the job with other firms. At n-firms, on the other hand, the wage offer function φ is relatively flat in this range, and encounters with other firms to not result in large wage increases.

We calculate that the variance of $\log(\omega_i)$, the log-wage residual, is 0.0056 overall, 0.0040 at n-firms, and 0.0122 at r-firms. When compared to the variance in log-ability, 0.1142, we find that search frictions in this population account for quite a small fraction of overall dispersion (4.71%).

5.1.3 Inefficient Mobility

One important implication of this model framework is that, when r and n firms compete for workers, an r-firm with a lower match value may win the worker simply because of the n-firm's unwillingness to negotiate. We call this phenomenon *inefficient mobility*.

Let $\theta_r < \theta_n$ denote the match values at an r-firm and n-firm, respectively. Now, suppose that $\varphi(\theta_n) < \theta_r$. Efficient mobility dictates that the worker must go to the firm with the greatest productive capacity, however in this case, since the highest wage available at that firm, $\varphi(\theta_n)$, is less than what the r-firm is willing to offer, inefficient mobility occurs.

Using the wage function φ derived from our structural parameter estimates, we show in Figure 9 the combinations of match values (x, y) that result in inefficient mobility. Furthermore, we can quantitatively evaluate the severity of this problem by computing the fraction of on-the-job encounters in steady state that result in an inefficient mobility decision. This has an analytic expression:

$$\text{Rate of inefficient mobility} = \int [F(\varphi^{-1}(x)) - F(x)] \cdot [(1-p)g_r(x) + pg_n(x)]dx \qquad (22)$$

Once again using our estimates, we apply this formula to calculate that 8.30% of on-the-job ecounters result in an inefficient mobility decision. We will revisit the preponderance of this phenomenon in our counterfactual exercises.

5.2 A Partial Equilibrium Counterfactual

To investigate the quantitative importance of the frequency of bargaining, p, to wage inequality, efficiency, and worker welfare, we conduct two counterfactual experiments. First, we eliminate bargaining altogether by setting $p = 0$. Second, we consider the changes in partial equilibrium when the fraction of bargaining firms is exogenously set at $p = 0.5$. In Table 6 we document some important aggregate statistics for these two counterfactual scenarios, relative to the baseline. Consistent with our previous arguments concerning wage inequality and renegotiating firms, we see that increasing (decreasing) the fraction of r-firms leads to increases (decreases) in wage dispersion due to search frictions. In fact, increasing the fraction of r-firms to 0.5 nearly doubles the variance of the log-wage residual. [10]

Two further important observations remain to be made. First, we note that increasing the fraction of r-firms leads to a marked increase in the rate of inefficient mobility: in the counterfactual equilibrium more than 20% of firm interactions result in a suboptimal mobility decision. This has consequences for average output per worker, which is reduced by 5 cents an hour. Though this may seem small, one should be reminded that this is a flow value for a single worker, which may still aggregate (across workers and over time) to a sizeable loss in output. On the other hand, we know that if *all* firms were type r, that there would be no inefficient mobility. Thus the mapping between p and the measure of inefficient moves is not monotonic.

Our second observation is that workers, according to average worker welfare in steady state, appear to prefer firms to negotiate less (lower values of p). Since n-firms, who trade-off profits with worker retention, are force through competition to offer higher wages, this is preferable in equilibrium to being at an n-firm where high wages must be solicited through renegotiation. Validation of this result can be found by returning to Figure 6, which shows that for a large range of match values, the implied bargaining power of workers at n-firms is much higher than for r-firms. In general, the answer to whether workers prefer a greater fraction of r-firms varies depending on the model's parameters. In particular, the worker's bargaining share, α, at r-firms, and the rate at which OTJ offers can be solicited, λ_e, critically determine the payoff to being at a renegotiating firm.

[10]Our sample consists only of individuals who have at most a high school education. Since Hall and Krueger find that individuals with higher levels of completed schooling are more likely to bargain over wages, this may be one factor in the much greater degrees of wage and earnings dispersion for college-completers in comparison with those who have lower levels of education.

6 General Equilibrium

Section forthcoming. See below for general details.

We first introduce the conditions under which p and the contact rates (λ_u, λ_e) are determined in equlibrium. We adopt the now standard model of vacancy posting[11], assuming that:

$$\lambda_u = \kappa^{\gamma}, \qquad \lambda_e = \mu_e \lambda_u, \qquad q = \kappa^{\gamma-1}$$

Where κ is market tightness, the ratio of searching workers to posted vacancies, and $q(\kappa)$ is the contact rate for firms with open vacancies searching for workers. Market tightness can be expressed as

$$\kappa = \frac{v_r + v_n}{M_u + \mu_e M_e}.$$

We follow Lise and Robin (2017) and assume that each firm type equates the marginal cost of vacancy creation with the marginal benefit of posting, such that:

$$-c_r'(v_r) + q(\kappa)\sum_a \int (1-\alpha)[S(a,\theta) - S(a,x)]^+ dF_\theta d\tilde{G}(x|a)\tilde{\pi}_a = 0 \tag{23}$$

$$-c_n'(v_n) + q(\kappa)\sum_a \int J\big(a,\theta,\varphi(\theta)\big)\mathbf{1}\{\varphi(\theta) > x\} dF_\theta d\tilde{G}(x|a)\tilde{\pi}_a = 0 \tag{24}$$

$$p = \frac{v_r}{v_r + v_n} \tag{25}$$

$$c_j(v_j) = \frac{c_j}{1+\psi} v_j^{1+\psi} \text{ for } j \in \{r, n\} \tag{26}$$

Where $\tilde{G}$ and $\tilde{\pi}$ are the endogenous distributions of best available offers and ability, conditional on having met this worker in the undirected pool of searchers. We combine these restrictions with our estimates to back out the parameters c_r, c_n, subject to a choice of the elasticities γ, ψ. We explore sensitivity of our counterfactual exercises to choices of these parameters.

Our chosen counterfactual exercise is an increase in the minimum wage, to \$10/hr.

7 Conclusion

Section forthcoming.

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[11] See Mortensen and Pissarides (1994) for a canonical application

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A Tables

Table 1: Descriptive Statistics from Hall and Krueger (2012) Survey

	High School, 21-30	All High School	All College
% Bargain	15.52	22.738	40.871
	[7.60, 25.89]	[19.565, 26.387]	[38.298, 44.366]
Mean Wage	12.73	14.367	22.888
	[10.94, 14.61]	[13.668, 15.129]	[22.193, 23.542]
Mean Wage | Bargain	17.36	14.735	25.007
	[8.35, 27.64]	[13.040, 16.602]	[23.658, 26.214]
Mean Wage | No Bargain	11.77	12.323	18.194
	[10.18, 14.13]	[11.640, 13.241]	[17.177, 19.113]
Sample Size	74	691	1822

Notes: This table shows some descriptive statistics from the survey data collected in Hall and Krueger (2012). Bracketed intervals indicate 95% confidence intervals for the statistics calculated. Bargained wages are judged by the answer to the survey question "When you were offered your current/previous job, did your employer make a "take-it-or leave-it" offer or was there some bargaining involved?" The left column shows statistics computed for high school graduates aged 21-30, the middle column shows high school graduates of all ages, the right column is for college graduates. Data is publically available at `http://www.stanford.edu/~rehall/Hall_Krueger_2011-0071_programs_and_results`

Table 2: Descriptive Statistics from SIPP Sample

Description	Notation	Moment
Average duration of unemployment spells	$\mathbb{E}[T_{i,s} \mid E_{i,s} = 0]$	5.847
		[5.527, 6.216]
Average duration of employment spells	$\mathbb{E}[T_{i,s} \mid E_{i,s} = 0]$	13.897
		[13.540, 14.278]
% Of (non-truncated) employment spells	$P[E_{i,s+1} = 1 \mid E_{i,s} = 1,\ s < S_i]$	31.825
ending in EE transition		[27.382, 35.116]
Average number of spells per worker	$\mathbb{E}[S_i]$	1.936
		[1.889, 2.014]
Average wage at beginning of sample	$\mathbb{E}[W_{i,0,0} \mid E_{i,0} = 1]$	11.239
		[10.986, 11.532]
% Unemployed at beginning of sample	$P[E_{i,0} = 0]$	20.632
		[18.815, 22.447]
Sample size	N	1488

Notes: This table shows decriptive statistics from the SIPP. Durations are reported in months, wages are reported in \$/hour. Bracketed intervals indicate 95% confidence intervals for the statistics calculated.

Table 3: Estimates

Model Parameters								
λ_u	λ_e	δ	p	α	b	σ_0	σ_1	π_θ
0.115	0.026	0.020	0.074	0.192	0.213	0.057	0.139	0.502
$[0.104, 0.123]$	$[0.023, 0.030]$	$[0.018, 0.022]$	$[0.058, 0.086]$	$[0.122, 0.221]$	$[0.212, 0.286]$	$[0.016, 0.057]$	$[0.065, 0.238]$	$[0.497, 0.504]$
Measurement Error				Ability Distribution				
$\sigma_{\epsilon,1}$	$\sigma_{\epsilon,2}$	π_ϵ		a_1	a_2	a_3	a_4	a_5
0.025	0.132	0.503		7.58	9.26	11.50	13.96	20.16
$[0.009, 0.038]$	$[0.111, 0.169]$	$[0.500, 0.507]$		$[7.31, 8.06]$	$[8.99, 11.52]$	$[9.56, 14.34]$	$[11.51, 14.51]$	$[19.03, 21.15]$
				π_1	π_2	π_3	π_4	π_5
				0.20	0.20	0.20	0.20	0.20

Notes: This table presents estimates from the MSM procedure of the baseline model, in which minimum wages do not bind. Parameters are as described in the text. F_θ, the match distribution, is modeled as a mixture of two normals with standard deviations (σ_1, σ_2) and mixing probability π_θ. Measurement error, F_ϵ is modeled similarly. Numbers in square brackets show the 95% confidence intervals for each parameter, which have been computed by nonparametric bootstrap, using 100 resamples of the data.

Table 4: Model Fit I: Transitions

Moment	Model	Data
$\mathbb{E}[T_{i,s} \mid E_{i,s} = 0]$	5.86	5.85
$\mathbb{E}[T_{i,s} \mid E_{i,s} = 1]$	13.90	13.90
$P[E_{i,s+1} = 1 \mid E_{i,s} = 1]$	0.31	0.32
P[Wage Bargained]	0.16	0.16

Table 5: Model Fit II: Distributions

	$\Delta \log(w) \mid EE$		$\Delta \log(w) \mid EUE$		$\Delta \log(w) \mid T_i = 24$		$\log(w)$	
	Model	Data	Model	Data	Model	Data	Model	Data
q_{10}	-0.12	-0.24	-0.24	-0.35	-0.09	-0.08	1.88	1.87
q_{20}	-0.06	-0.06	-0.16	-0.21	0.00	0.02	1.98	2.00
q_{30}	-0.01	0.00	-0.11	-0.09	0.04	0.05	2.09	2.08
q_{40}	0.02	0.00	-0.06	-0.04	0.07	0.07	2.19	2.20
q_{50}	0.05	0.06	-0.02	0.00	0.10	0.10	2.30	2.30
q_{60}	0.09	0.12	0.02	0.05	0.13	0.12	2.40	2.40
q_{70}	0.13	0.17	0.06	0.12	0.16	0.16	2.51	2.52
q_{80}	0.17	0.22	0.11	0.20	0.22	0.22	2.68	2.67
q_{90}	0.24	0.36	0.19	0.36	0.32	0.37	2.88	2.88

Table 6: Results for Partial Equilibrium Experiments

	Baseline ($p = 0.074$)	$p = 0$	$p = 0.5$
$\mathbb{E}[W \mid n - \text{firm}]$	11.08	11.15	10.65
$\mathbb{E}[W \mid r - \text{firm}]$	10.56	NaN	10.88
Worker Welfare	148.56	150.39	136.52
$\mathbb{V}[\log(\omega)]$	0.0056	0.0041	0.0117
$\mathbb{V}[\log(\omega) \mid n - \text{firm}]$	0.0040	0.0041	0.0049
$\mathbb{V}[\log(\omega) \mid r - \text{firm}]$	0.0122	NaN	0.0142
Average output per worker ($/hr)	11.04	11.08	10.99
Rate of inefficient mobility (%)	8.30	0.00	20.50

Table 7: Model Estimates for $p = 0$ and $p = 1$

	Baseline Model	No Renegotiation ($p = 0$)	All Renegotiation ($p = 1$)
λ_u	0.115	0.118	0.122
λ_e	0.026	0.026	0.021
δ	0.020	0.020	0.020
p	0.074	0	1
α	0.192	-	0.097
b	0.213	0.263	0.572
σ_0	0.057	0.182	0.191
σ_1	0.139	0.010	0.005
π_θ	0.502	0.502	0.499
$\sigma_{m,0}$	0.025	0.045	0.005
$\sigma_{m,1}$	0.132	0.238	0.151
π_m	0.503	0.506	0.505
θ^*	0.787	0.865	0.838
Q_N	0.061	0.394	0.089

Table 8: Model Fit for $p = 0$ and $p = 1$: Transitions

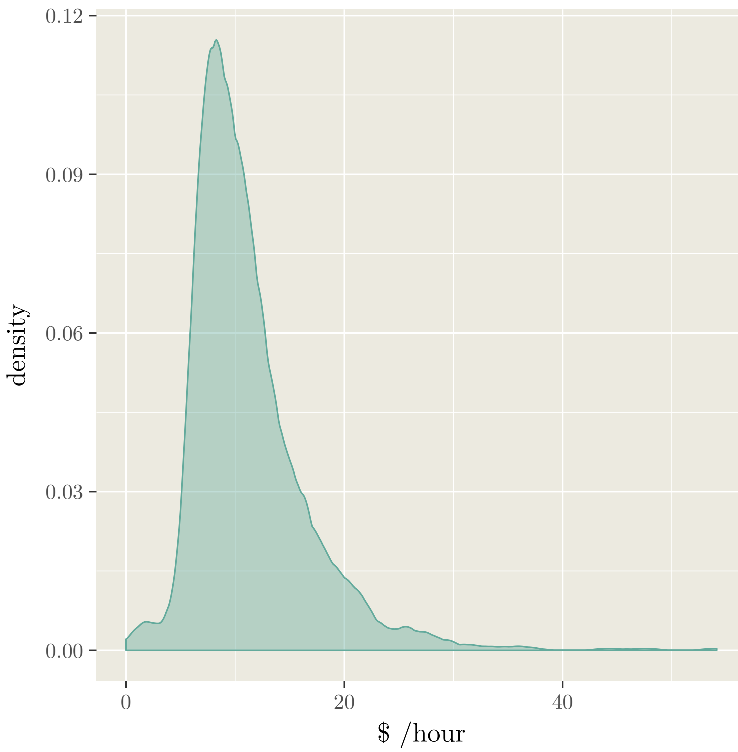
Moment	$p = 0$	$p = 1$	Data
$\mathbb{E}[T_{i,s} \mid E_{i,s} = 0]$	5.85	5.83	5.85
$\mathbb{E}[T_{i,s} \mid E_{i,s} = 1]$	13.89	13.82	13.90
$P[E_{i,s+1} = 1 \mid E_{i,s} = 1]$	0.32	0.31	0.32

Table 9: Model Fit for $p = 0$ and $p = 1$: Distributions

	$\Delta \log(w) \mid EE$			$\Delta \log(w) \mid EUE$			$\Delta \log(w) \mid T_i = 24$		
	$p = 0$	$p = 1$	Data	$p = 0$	$p = 1$	Data	$p = 0$	$p = 1$	Data
q_{10}	-0.25	-0.09	-0.24	-0.33	-0.26	-0.35	0.00	-0.11	-0.08
q_{20}	-0.11	-0.01	-0.06	-0.19	-0.17	-0.21	0.00	-0.01	0.02
q_{30}	-0.03	0.04	0.00	-0.12	-0.12	-0.09	0.00	0.03	0.05
q_{40}	0.01	0.08	0.00	-0.06	-0.07	-0.04	0.00	0.07	0.07
q_{50}	0.05	0.11	0.06	-0.01	-0.02	0.00	0.00	0.10	0.10
q_{60}	0.09	0.13	0.12	0.03	0.00	0.05	0.00	0.13	0.12
q_{70}	0.15	0.16	0.17	0.09	0.02	0.12	0.00	0.15	0.16
q_{80}	0.23	0.21	0.22	0.17	0.07	0.20	0.00	0.21	0.22
q_{90}	0.35	0.29	0.36	0.30	0.15	0.36	0.00	0.30	0.37

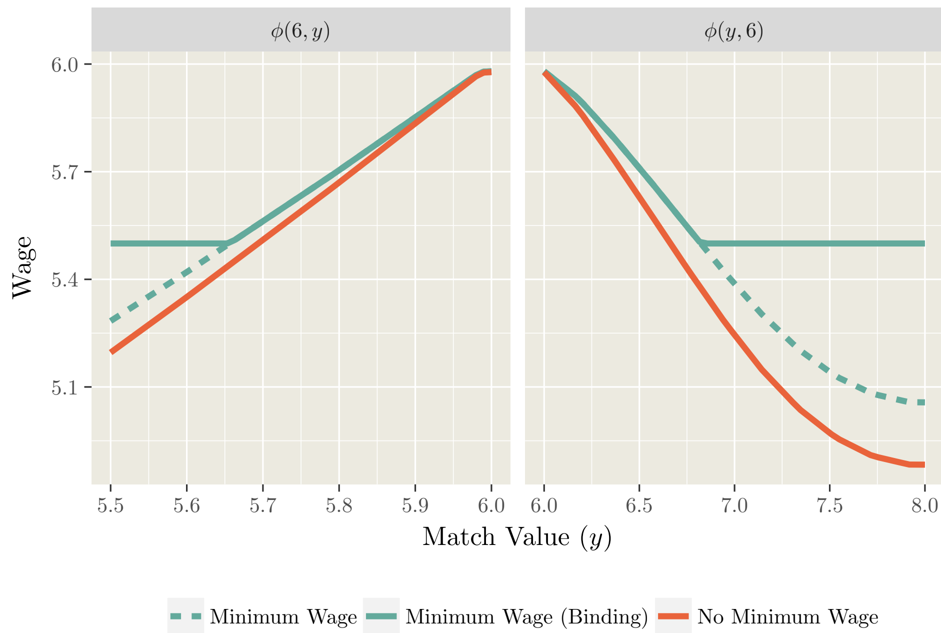
B Figures

Figure 1: Steady State Wage Distribution from the SIPP



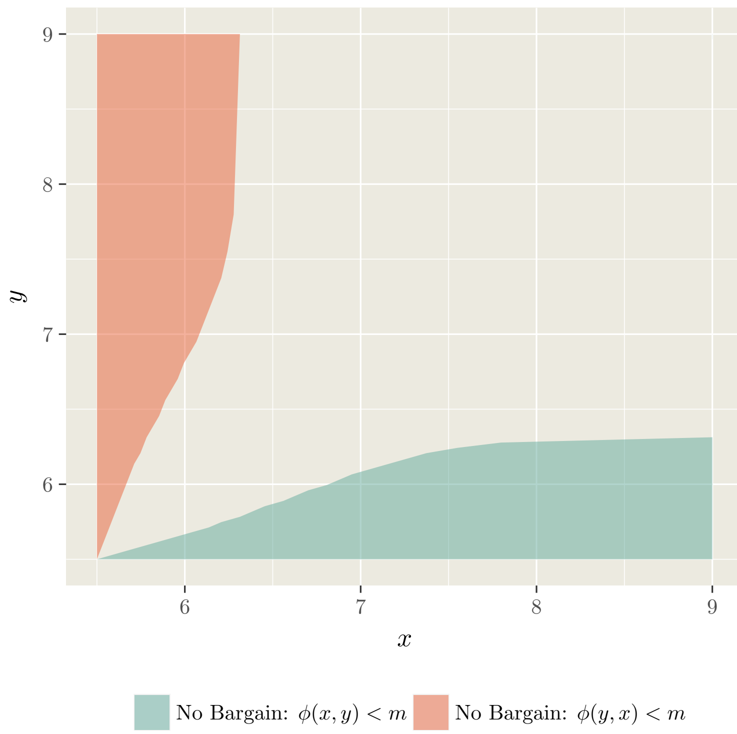
Notes: This figure shows a nonparametric density plot of "steady state" wages, which are taken from workers who are employed at the begnning of our 24-month SIPP sample.

Figure 2: Binding Minimum Wages at r-Firms



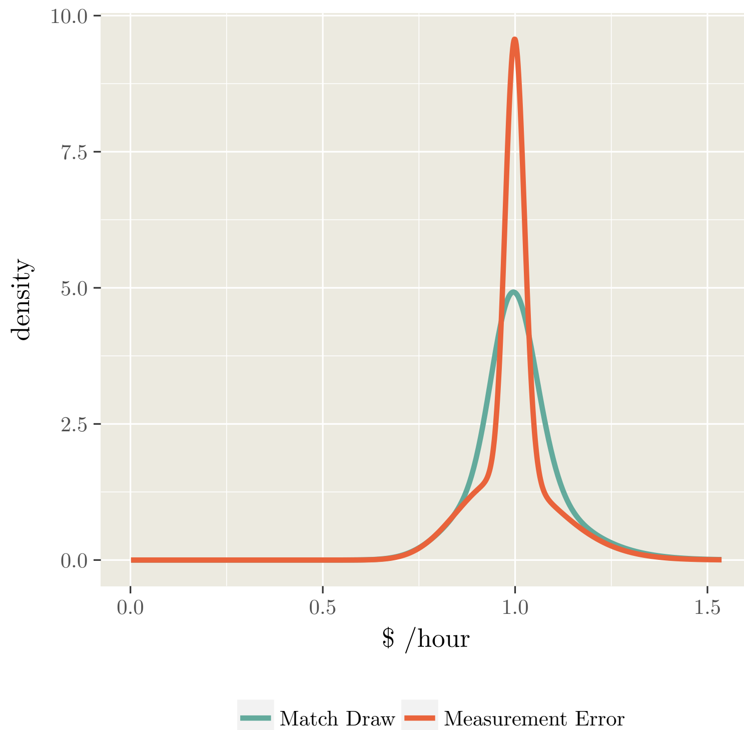
Notes: In this figure we plot bargained wages ϕ. In the left panel, we fix the outside option at a match value of \$6/hr and vary the outside option y. On the right, we fix the outside option at \$6/hr and vary the winning match y.

Figure 3: No Bargain Zones when minimum wage binds



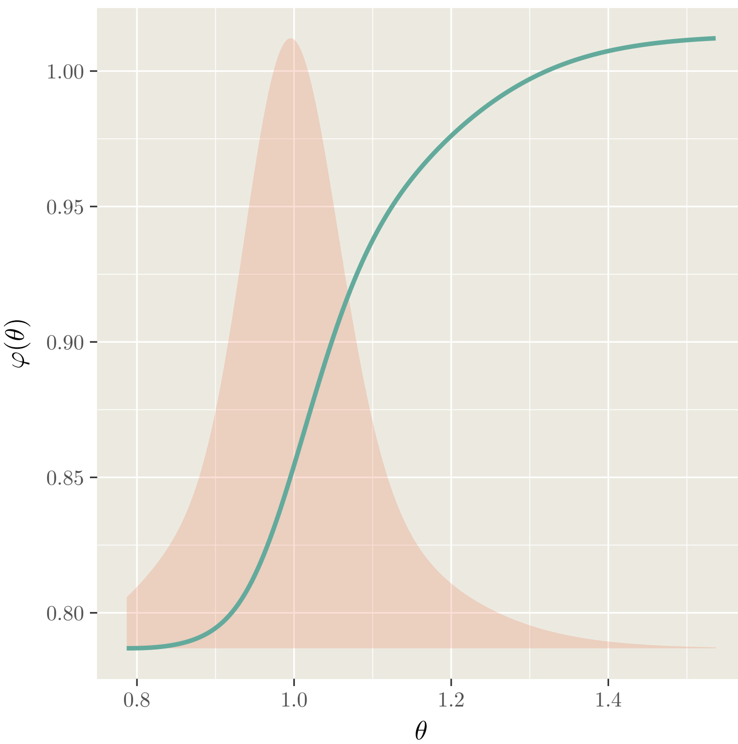
Notes: This figure shows the combinations of x and y for which the minimum wage interferes with the bargaining process.

Figure 4: Densities of the match distribution, F_θ, and measurement error, F_ϵ



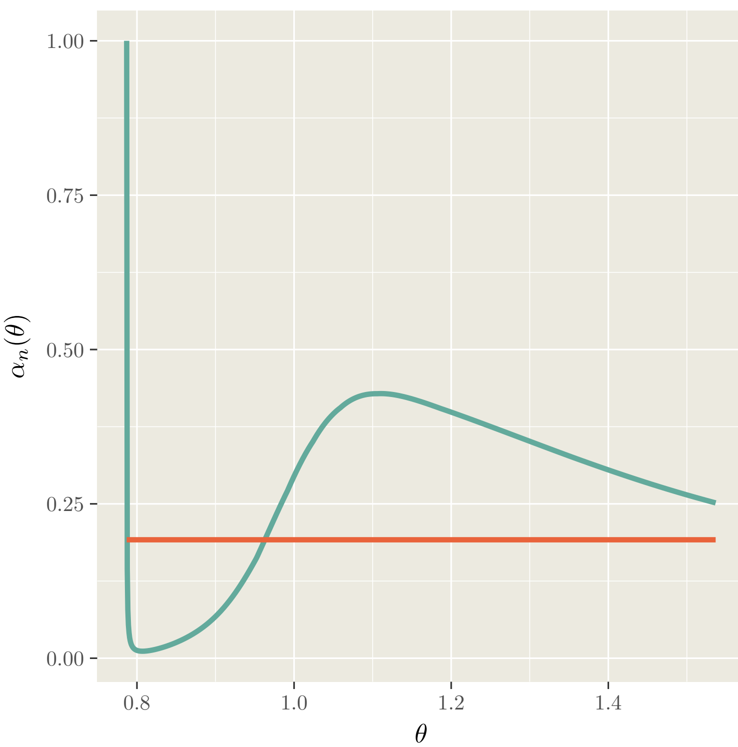
Notes: This figure shows the the density of the distribution of match values, F_θ, and the density of the measurement error distribution, F_ϵ. Realizations of both random variables can be measured in $/hr.

Figure 5: Wage offer function, φ, and density of match values, f_θ



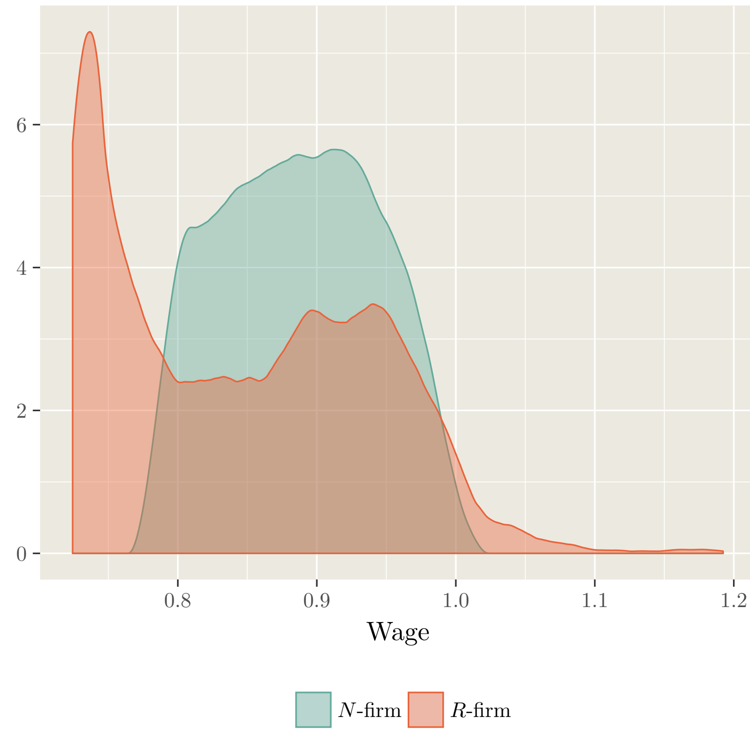
Notes: This figure shows the wage offer function, φ of n-firms. For exposition, the (re-scaled) density f_θ of the match distribution F_θ is shown in the background.

Figure 6: Implied Bargaining Parameters for n-firms, $\alpha_n(\theta)$.



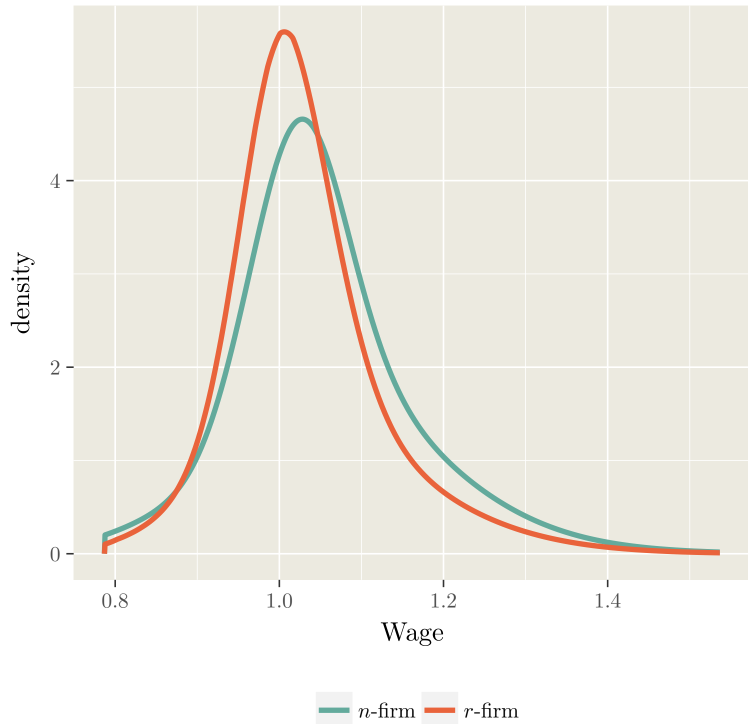
Notes: This figure shows the implied bargaining parameter, defined as the share of surplus to the worker given the wage offer, $\varphi(\theta)$ of n-firms. For reference, the estimated bargaining parameter for r-firms, α, is plotted also. See equation (21) for further explanation.

Figure 7: Wage distribution in steady state at r and n firms



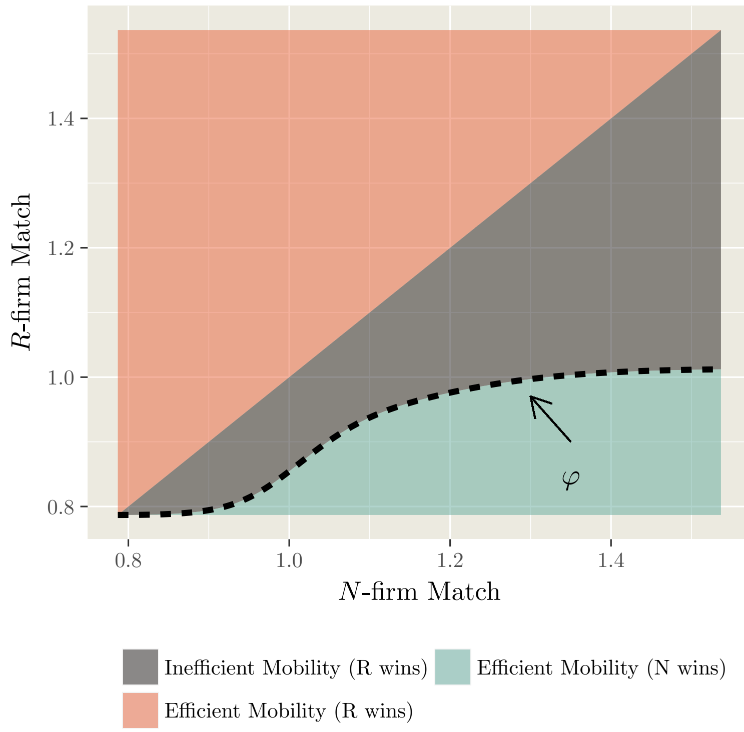
Notes: This figure shows the wage distribution in steady state at n and r firms, using parameter estimates from the baseline model.

Figure 8: Densities of match values in steady state at r and n firms



Notes: This figure shows the distribution in steady state of match values at n and r firms, using parameter estimates from the baseline model.

Figure 9: Inefficient Mobility



Notes: This figure shows the combinations of matches at r and n firms that result in efficient and inefficient mobility. An n-firm with match x wins if and only if the wage offer $\varphi(x)$ is greater than the r-firm's match y. When $\varphi(x) < y < x$, the model exhibits inefficient mobility.

C Proofs

In the following set of results, we make extensive use of equations (13) and (15), which express the n-firm's value function as:

$$J_n(\theta, w) = \Gamma(\Phi(w), F(w))(\theta - w) = \Gamma(w)(\theta - w).$$

The second equality is notationally abusive, but useful, so we adopt it here. The function Γ is an expression that combines the probability that w successfully hires a worker, with the rate at which the worker is lost when w is the non-negotiable wage.

Secondly, we assume the following tie-breaking rule: when two-firms make equally valuable wage offers, the worker moves from the incumbent to the new firm. It should be noted that the results below are robust to the alternative rule: that the worker defaults to the incumbent firm. Furthermore, in equilibrium, ties occur with zero probability.

Proof of Lemma 1

Proof. Since all offers $w < \theta^*$ are, by definition, never accepted, we know that $\underline{w} \geq \theta^*$. Now assume that $\underline{w} > \theta^*$, and consider the optimal offer made by a firm when a match $x \in (\theta^*, \underline{w})$ is drawn. Since any offer $w \in (\theta^*, x)$ is both profitable to the firm and acceptable to an unemployed worker (who is met with positive probability), we have a contradiction. $\square$

Lemma 2. Γ *is (i) strictly increasing; and (ii) continuous if and only if* Φ *is continuous.*

Proof. **(i)**: This follows directly from our assumption that F is strictly increasing in w (there are no gaps in the distribution of match qualities) and Φ is, by definition, non-decreasing. Thus, Γ must be strictly increasing in w, given its form in (15).

(ii): This is elementary, since we see that Γ is a continuous transformation of Φ and F. $\square$

Lemma 3. *In equilibrium, the wage offer distribution* Φ *is continuous.*

Proof. Note that a discontinuity in Φ at some w implies a mass point at w and, by Lemma 2, Γ is discontinuous. Given the tie-breaking rule, we have that $\lim^+ \Gamma(w) > \Gamma(w)$. This is caused by a discontinuous increase in the probability of retaining a worker.[12] Hence, $\lim^+ J(\theta, w) > J(\theta, w)$ for any θ, and for any firm offering wage w, an improvement in profit can be made by offering $w + \epsilon$ where ϵ is arbitrarily small. Thus no firm prefers to offer w, a contradiction. $\square$

The following corollary is immediate.

Corollary 1. Γ *is continuous.*

Lemma 4. *In equilibrium, wages are given by an almost everywhere deterministic function,* φ.

Proof. Suppose otherwise. Then for a firm with match θ, the firm is indifferent over a set $\mathcal{W}$ with positive Lebesgue measure:

$$\Gamma(w)(\theta - w) = c, \;\forall\; w \in \mathcal{W}.$$

Likewise, for a firm with match $\hat{\theta} \neq \theta$, indifference is achieved over a set $\hat{\mathcal{W}}$:

$$\Gamma(w)(\hat{\theta} - w) = \hat{c}, \;\forall\; w \in \hat{\mathcal{W}}.$$

If $\mathcal{W} \cap \hat{\mathcal{W}}$ has positive measure, we must have $\Gamma(w)(\theta - \hat{\theta}) = c - \hat{c}$ for all w in this intersection, which can be true only if $\Gamma(w)$ is everywhere constant, contradicting Lemma 2. Therefore, $\mathcal{W} \cap \hat{\mathcal{W}} = \emptyset$, and so this can only be true for a countable set of matches, which have measure zero under our regularity assumptions on F.

$\square$

[12] Notice that if we had assumed the alternative tie-breaking rule, there would be a discontinuous increase in the probability of hiring the worker, and the result would still follow.

Lemma 5. *The wage offer function, φ, is strictly increasing in match values, θ.*

Proof. Let $\varphi(\theta) = w$. This implies that:

$$\Gamma(w)(\theta - w) > \Gamma(\hat{w})(\theta - \hat{w}),\ \forall \hat{w} < w$$

Rearranging this expression we get:

$$(\Gamma(w) - \Gamma(\hat{w}))\theta > \Gamma(w)w - \Gamma(\hat{w})\hat{w},\ \forall \hat{w} < w$$

By Lemma 2, $\Gamma(w) - \Gamma(\hat{w}) > 0$, which implies that for any $\theta' > \theta$, we have

$$(\Gamma(w) - \Gamma(\hat{w}))\theta' > \Gamma(w)w - \Gamma(\hat{w})\hat{w},\ \forall \hat{w} < w$$

So when the match value is θ', the above inequality implies that w is also preferred to all $\hat{w} < w$, and so $\varphi(\theta') \geq \varphi(\theta)$, However, if this inequality is not strict, repeated application of the above inequality implies that $\varphi(z) = w$ for all $z \in [\theta, \theta']$. However, this implies a discontinuity in Φ, contradicting Lemma 3. Thus, the inequality must be strict. $\square$

To prove differentiability, we make use of the following commonly known result.

Lemma 6. *If a function, $f : \mathbb{R} \mapsto \mathbb{R}$, is bounded, and monotonically increasing, it is almost everywhere (according to Lebesgue measure) differentiable.*

Proof. See, for example, Result 11.42 in Titchmarsh (1932). $\square$

Lemma 7. *The wage offer function, φ is almost everywhere differentiable and lower semi-continuous.*

Proof. Consider the function φ on the domain $[\theta^*, \theta]$. Since $\varphi(\theta)$ is bounded above by θ, bounded below by θ^*, and strictly monotonically increasing, it follows from Lemma 6 that φ must be almost everywhere differentiable (and hence almost everywhere continuous).

Consider now a potential discontinuity in φ at θ. Let d^+ and d^- denote the differentiation operation, taking *right* and *left* limits, respectively. Let $\varphi^-(\theta) = w_0$ and $\varphi^+(\theta) = w_1$. We know that $w_0 < w_1$. A discontinuity in φ implies that the distribution Φ is flat over the range $[w_0, w_1]$, and hence: $d^+\Phi(w_0) = d^-\Phi(w_1) = 0$. Supposing that $\varphi(\theta) = w_1$ (and hence the function is upper semicontinuous), optimality of this wage choice implies that the pair of inequalities

$$d^+J(\theta, w_1) \leq 0, \qquad d^-J(\theta, w_1) \geq 0$$

must hold. Taking left and right derivatives at this point gives inequalities

$$\frac{\lambda_1(\rho + 2\Psi(w_1))(pf(w_1) + (1-p)d^+\Phi(w_1))}{(\rho + \Psi(x))\Psi(x)} - 1 \geq 0$$
$$\frac{\lambda_1(\rho + 2\Psi(w_1))(pf(w_1) + 0)}{(\rho + \Psi(x))\Psi(x)} - 1 \leq 0$$

Since $d^+\Phi(w_1) = \phi(w_1) > 0$, one inequality here contradicts the other. Hence, φ must be lower semi continuous (application of the above inequalities at w_0 yields no such contradiction). $\square$

- These continuities are essentially kink points in the overall steady state distribution, flat points in Φ.
- This suggests an algorithm for solution: at each point, check to see if there is an equally profitable wage above, assuming no n-firm posts in between.
- If this holds, then the sufficient condition for optimality of this wage is violated in eqm.

D Model Solution

D.1 Solving the Steady State

We first derive the distribution of best attainable offers, G, of employed workers across this state by balancing the flow equation:

$$dG(x) = -(\delta + \lambda_e p\overline{F}(x) + \lambda_e(1-p)\overline{\Phi}(x))G(x)M_e + \lambda_u[p(F(x) - F(\theta^*)) + (1-p)(\Phi(x) - F(\theta^\star))]M_u$$

Setting $dG(x) = 0$ and rearranging gives the steady state distribution as:

$$G(x) = \frac{p(F(x) - F(\theta^*)) + (1-p)(\Phi(x) - \Phi(\theta^\star))}{\delta + \lambda_e p\overline{F}(x) + \lambda_e(1-p)\overline{\Phi}(x)} \frac{\lambda_u M_u}{M_e}$$

It will be helpful to substitute the expression:

$$\Psi(x) = \delta + \lambda_e p\overline{F}(x) + \lambda_e(1-p)\overline{\Phi}(x)$$

such that:

$$G(x) = \frac{p(F(x) - F(\theta^*)) + (1-p)(\Phi(x) - F(\theta^\star))}{\Psi(x)} \frac{\lambda_u M_u}{M_e}.$$

$\Psi(x)$ is the exit rate at a firm where the maximum attainable wage is x. Next, let $G_r(x)$ indicate the measure of workers at r-firms with match value x, and let $G_n(x)$ indicate the measure of workers at n-firms with wage x, such that $G_r(x) + G_n(x) = G(x)$. We can derive the flow equations:

$$dg_r(x) = -[\delta + \lambda_e p\overline{F}(x) + \lambda_e(1-p)\overline{\Phi}(x)]g_r(x)M_e + pf(x)[\lambda_u M_u + \lambda_e M_e G(x)] \tag{27}$$

$$dg_n(x) = -[\delta + \lambda_e p\overline{F}(x) + \lambda_e(1-p)\overline{\Phi}(x)]g_n(x)M_e + (1-p)\phi(x)[\lambda_u M_u + \lambda_e M_e G(x)] \tag{28}$$

Subsituting the derived expression for G and imposing a blanced flow steady state yields:

$$g_r(x) = \frac{\lambda_u pf(x)(\delta + \lambda_e\overline{F}(\theta^*)}{\Psi(x)^2} \frac{M_u}{M_e} \tag{29}$$

$$g_n(x) = \frac{\lambda_u(1-p)\phi(x)(\delta + \lambda_e\overline{F}(\theta^*)}{\Psi(x)^2} \frac{M_u}{M_e} \tag{30}$$

Finally, to derive the distribution of wages at renegotiating firms, we think about the conditional distribution of workers at a firm with match x whose last best offer had value q. The flow equation for the mass of workers of this type is:

$$d(H(q|x)g_r(x)) = -(\delta + \lambda_e p\overline{F}(x) + \lambda_e(1-p)\overline{\Phi}(x))H(q|x)g_r(x)M_e + \lambda_e pf(x)G(q)M_E + \lambda_u pf(x)M_U$$

Once again, we can substitute our expressions for g_r and G, impose that the steady-state flow is equal to zero, and rearrange to get:

$$H(q|x) = \left(\frac{\Psi(x)}{\Psi(q)}\right)^2$$

D.2 Solving the Wage Equation, φ

In the main text, we derived a condition such that each wage offer, $\varphi(\theta)$, solves the first order condition for a firm at each match value θ, given the local shape of φ at θ. We can rearrange this condition, in equation (17), to get a first order differential equation:

$$\varphi'(\theta) = \frac{\Gamma_1\big(F(\theta), F(w)\big)f(\theta)}{\Gamma\big(F(\theta), F(w)\big)/(\theta - w) - \Gamma_2\big(F(\theta), F(w)\big)f(w)} \tag{31}$$

$$\Gamma_1\big(F(\theta), F(w)\big) = \frac{\lambda_e(1-p)(\rho + 2\text{exit}(\theta, w))}{(\text{exit}(\theta, w)(\rho + \text{exit}(\theta, w))^2} \tag{32}$$

$$\Gamma_2\big(F(\theta), F(w)\big) = \frac{\lambda_e p(\rho + 2\text{exit}(\theta, w))}{(\text{exit}(\theta, w)(\rho + \text{exit}(\theta, w))^2} \tag{33}$$

$$\text{exit}(\theta, w) = \delta + \lambda_e(p\overline{F}(w) + (1-p)\overline{F}(\theta)) \tag{34}$$

The term (θ, w) is the rate at which a worker at an n-firm with match θ and wage w will leave the firm, in equilibrium. Two important insights can be made when inspecting equation (31). First, we see that the first term in the denominator explodes when the wage offer w is close to the match value θ. Since Lemma 1 requires that the limit of $\varphi(\theta)$ as $\theta \to \theta^*$ is equal to θ^*, we know that φ must be increasingly flat in the region close to the reservation match value θ^*. Second, we see that the density of the match distribution $f_\theta(\theta)$ appears in the numerator, suggesting that φ will be steeper when the density is high, and flatter when the density is smaller (say, in the tails of the distribution). Both of these predictions are borne out numerically.

One issue in using the differential equation above is that Proposition 1 does not guarantee that φ is everywhere continuous, and the first-order condition is known only to be necessary and not sufficient. The algorithm we use accounts for potential discontinuities in φ by globally checking for optimality at each step.

To do this, we need to use the following profit function, which gives the profit to the firm under the equilibrium condition that wage offers are ranked according to θ.

$$J^*(\theta, w) = \frac{\theta - w}{\text{exit}(\theta, w)(\rho + \text{exit}(\theta, w))} \tag{35}$$

The algorithm proceeds as follows, given a predetermined grid $\{\theta_0, \theta_1, ..., \theta_J\}$ with $\theta_0 = \theta^*$. To initialize the algorithm, we set $w_0 = \theta^*$:

1. Given θ_{j-1}, $w_{j-1}(= \varphi(\theta_{j-1}))$, use (31) and either Euler's method or a more advanced method such as Runge-Kutta to compute the step $d\varphi_j$.

2. Check for global optimality by solving $w^* = \arg\max_{w \in [w_{j-1}, \theta_j]} J^*(\theta_j, w)$.

3. If $w^* > w_{j-1}$, set $w_{j+1} = w^*$.

4. Otherwise, set $w_j = w_{j-1} + d\varphi_j$.

The idea here is that, if $w^* > w_{j-1}$, then the shape of the match distribution F supports a discontinuity at θ_j, such that no firm offers between w_{j-1} and w^*. In addition, the marginal firm θ_j is indifferent between these wage offers. If, on the other hand, the firm prefers to offer w_j (the lowest wage available) then we must introduce marginal wage competition by way of $\varphi'(\theta)$.

D.3 Solving Surplus Equations and r-Firm wages

In this section we provide further details for solving the surplus equation, S, which defines values at both r-firms and n-firms. For ease of exposition, we provide once more the recursive definition of S:

$$(\rho + \delta)S(x) = x + \lambda_e p \int_x \alpha(S(y) - S(x))dF + \lambda_e(1-p)\int_{\varphi^{-1}(x)} (S(\varphi(y)) - S(x))dF + \delta V_u$$

If we differentiating the surplus equation and rearranging gives:

$$S'(x) = \frac{1}{\rho + \delta + \lambda_e(\alpha p \overline{F}(x) + (1-p)\overline{\Phi}(x)}$$

bearing in mind that $\Phi(x) = F(\varphi^{-1}(x))$. This, in turn, permits us to write:

$$S(x) = S(\theta^*) + \int_{\theta^*}^{x} \frac{1}{\rho + \delta + \lambda_e(\alpha p \overline{F}(z) + (1-p)\overline{\Phi}(z))} dz = S(\theta^*) + \hat{S}(x)$$

In fact, adapting the method proposed by Cahuc et al. (2006), integration by parts yields the following analytic solution:

$$(\rho + \delta)S(x) = x + \lambda_e \int_x \frac{\alpha p F(z) + (1-p)\Phi(z)}{\rho + \delta + \lambda_e(\alpha p \overline{F}(z) + (1-p)\overline{\Phi}(z)} dz + \delta S(\theta^*)$$

In practice, we solve the model by linearly interpolating $\hat{S}$ over grid points in the space for θ. In addition, the wage equation ϕ can be written as:

$$\begin{aligned}\phi(x,y) = (\rho + \delta + \lambda_e p \overline{F}(y) + \lambda_e(1-p)\overline{\Phi}(y))(\alpha\hat{S}(x) + (1-\alpha)\hat{S}(y)) - \rho S(x^*) \\ - \lambda_e p\left[\int_y^x [(1-\alpha)\hat{S}(z) + \alpha\hat{S}(x)]dF(z) + \int_x [(1-\alpha)\hat{S}(x) + \alpha\hat{S}(z)]dF(z)\right] \\ - \lambda_e p\left[\int_{\varphi^{-1}(y)}^{\varphi^{-1}(x)} [(1-\alpha)\hat{S}(\varphi(z)) + \alpha\hat{S}(x)]dF(z) + \int_{\varphi^{-1}(x)} \hat{S}(\varphi(z))dF(z)\right] \quad (36)\end{aligned}$$

Alternatively, using the restriction that $V(x,y) = (1-\alpha)S(y) + \alpha S(x)$, algebra yields the following expression for wages:

$$\phi(x,y) = (1-\alpha)y + \alpha x - \lambda_e p(1-\alpha)^2\left[\int_y^x S(z)dF(z) + \overline{F}(x)S(x) - \overline{F}(y)S(y)\right] \quad (37)$$

The third term in this expression signifies the extent to which a worker is compensated for lower wages today with the promise of future appreciation in wages. This, in turn, depends critically on the proportion, p, of firms that are willing to bargain. Finally, we can also solve for the value of unemployment as:

$$\rho S(\theta^\star) = b + \lambda_u \int_{\theta^\star} (p\alpha\hat{S}(x) + (1-p)\hat{S}(\varphi(x)))dF(x)$$

Similarly the surplus equation at $\theta^\star$ is

$$\rho S(\theta^\star) = \theta^\star + \lambda_e \int_{\theta^\star} (p\alpha\hat{S}(x) + (1-p)\hat{S}(\varphi(x)))dF(x)$$

Combining these two expressions is sufficient to pin down $\theta^\star$. This concludes our practical discussion of how to solve the model in equilibrium.

E Data and Sample Construction

Section forthcoming.