Are We Undercounting Reallocation's Contribution to Growth?

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Abstract

Three recent studies examining the sources of aggregate productivity growth in India's manufacturing sector use two different methodologies and find that it was driven by within-plant increases in technical efficiency more than between-plant reallocation of inputs. This is surprising given the nature of the economic reforms in India, where many barriers to input reallocation were removed. In this paper we show that these findings of a limited role for reallocation in growth may be an artifact of the way these studies estimate reallocation. Using microdata on manufacturing from 4 countries — the U.S., Chile, Colombia, and Slovenia — we show that ignoring the reallocation of intermediate inputs significantly understates the contribution of reallocation to aggregate productivity growth while overstating technical-efficiency's role. The second approach followed by the researchers uses average products as a proxy for marginal products in reallocation measurement. We show that in these four countries using average products instead of marginal products significantly underestimates the contribution of reallocation to growth.

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1 Introduction

Many reforms introduced in recent decades in India were largely aimed at correcting allocative inefficiencies. Following these reforms India experienced a robust increase in aggregate productivity growth of almost 5% per annum starting in the early 1990s. Three recent studies looking to quantify the contribution of the reallocation of inputs to this growth all find that it played very little role, and that growth was instead driven by within-plant gains in technical efficiency (Sivadasan (2009), Harrison, Martin, and Nataraj (2011) (HMN), Bollard, Klenow, and Sharma (2013)). For example, Bollard, Klenow, and Sharma (2013) (BKS) remark "As many economists believe Indian reforms during this era improved resource allocation, the absence of a growth pickup from reallocation is surprising," calling it "India's Mysterious Manufacturing Miracle."

The result is also surprising because two very distinct approaches were taken by these researchers in their search for reallocation growth. The approach in BKS uses a value added (VA) production function to estimate input marginal products and then tracks movements of inputs across plants with differing marginal products for the same input, similar in spirit to the definitions of reallocation from Basu and Fernald (2002) and Petrin and Levinsohn (2012). Both HMN and Sivadasan follow the Olley and Pakes (1996) decomposition to estimate the contribution of reallocation to growth. They estimate gross output (GO) production functions to recover technical efficiency residuals - the average product of all inputs taken together at a plant. Reallocation's contribution is then the sum (across all plants) of these average products weighted by changes in output shares.[1]

In this paper we show that these findings of a limited role for reallocation in growth may be an artifact of the way these studies estimate reallocation. In the case of BKS, when one uses a value-added production function, reallocation growth coming from intermediate inputs is misclassified as technical efficiency growth, a theoretical point noted by Basu and Fernald (1997). While we do not have access to the Indian data, we investigate the empirical relationship between estimated reallocation growth using manufacturing data from the U.S., Chile, Colombia, and Slovenia. In the U.S. manufacturing data, when we use VA production functions (as BKS does), we estimate technical efficiency growth of 0.77% per annum and reallocation growth of 1.13% per annum from

[1] More precisely, Olley-Pakes uses the average product relative to the unweighted industry average.

1976-1996. Using gross output production functions, we estimate technical efficiency growth of -0.54% per annum and reallocation growth of 2.15% per annum, so reallocation growth is understated by 1.02% when one uses VA production functions on the U.S. data. In terms of the relative roles in growth in Chile the VA production function understates (overstates) growth from reallocation (technical-efficiency) on average by 1.87% per annum from 1979-1996. In Colombia the VA production function understates (overstates) growth from reallocation (technical-efficiency) on average by 1.24% per annum from 1977-1991. In Slovenia reallocation of intermediates makes a relatively small contribution to aggregate productivity growth in manufacturing (compared to the other 3 countries) so the gross output and VA measures of reallocation differ by only 0.10% per year from 1994-2004.

In the HMN and Sivadasan papers they use the Olley-Pakes definition of reallocation, where average products proxy for marginal products of inputs when tracking reallocation growth. Below we use a simplified version of the Hsieh and Klenow (2009) setup to illustrate that the reallocation as defined in Olley-Pakes can be *negatively* correlated with the change in final demand due to the reallocation of inputs. We then show that when we apply the Olley-Pakes (OP) measurement to U.S. manufacturing data, the OP measure understates reallocation's contribution by 4.64 % per year. Census data from Colombia shows that OP reallocation was about 1.90% per year whereas the marginal product definition of reallocation reports and average 3.54% growth per annum from reallocation. Aggregate productivity growth from reallocation was *negative* and close to zero for Chile and Slovenia (-0.34% and -0.02% per year, respectively) when one uses the OP definition but Chile and Slovenia respectively have 3.09% and 1.89% per annum growth from reallocation using the marginal product definition.

1.1 Gross-Output vs. Value-Added Production Functions

We now show how value-added production functions count all reallocation from intermediate inputs as technical efficiency growth. Consider a continuous-time single-good economy with one producer that uses her own output as either an (intermediate) input into production or as final consumption. Let Q denote gross output and let $C = Q - M$ denote the amount of output left for consumption after Munits of it are used in production, and let utility $U(c) = c$. The production function is given as

$$Q = f(M, \omega)$$

with ω denoting technical efficiency. The price of the input/output is normalized to one and the optimal allocation between consumption and intermediate use is the M that satisfies $\frac{\partial f(M,\omega)}{\partial M} = 1$, where the (value of the) marginal product is equated with the input price.

At any instant the additional output going to consumption is given by $dC = dQ - dM$. Totally differentiating the production function and plugging in then gives

$$dC = \frac{\partial f}{\partial M} dM + \frac{\partial f}{\partial \omega} d\omega - dM = \frac{\partial f}{\partial \omega} d\omega + (\frac{\partial f}{\partial M} - 1) dM. \quad (1)$$

When $M$is allocated optimally all growth comes from increases in technical efficiency $\frac{\partial f}{\partial \omega} d\omega$. In this case the value added production function deducts from the value of output Q the value of the intermediate input used in production M, and the change in the value added production function, given by $dV = dQ - dM$, is exactly equal to the change in output dC. However, if market imperfections lead to a wedge between the marginal product and the input price, so $\frac{\partial f(M,\omega)}{\partial M} > 1$, then the value added production function is going to mistakenly count growth arising from intermediates being reallocated from production to consumption as growth from increases in technical efficiency. Technical efficiency will be overstated by $(\frac{\partial f}{\partial M} - 1)dM$.

1.2 Marginal Product vs. Average Product

The Olley-Pakes index of aggregate productivity growth (and similar indexes such as Bailey, Hulten and Campbell 1992) uses a definition of productivity growth that is not directly based on changes in industry value added, as is traditionally done. Instead it is based on looking at the change in share-weighted average products in the industry:

$$\sum_i s_{it}\omega_{it} - \sum_i s_{it-1}\omega_{it-1}$$

where ω_{it} is the estimated technical efficiency residual at plant i at time t. This leads to the OP decomposition of $\sum_i s_{it}\omega_{it} - \sum_i s_{it-1}\omega_{it-1}$ which is given as

$$\Delta\bar{\omega_t} + [\sum(s_{it} - \bar{s}_t)(\omega_{it} - \bar{\omega_t}) - \sum(s_{it-1} - \bar{s}_{t-1})(\omega_{it-1} - \bar{\omega_{t-1}})].$$

The first term is the change in the *unweighted* averages of technical efficiency at time t minus the change in *unweighted* averages of technical efficiency at time t-1 and is referred to as the "technical efficiency" term. The term in brackets — the covariance term — is sometimes interpreted as the reallocation term.

We use a simplified form of the Hsieh-Klenow setup to show that reallocation growth measurement based on average products can be a poor proxy for marginal products. Consider a single-good economy with two plants that convert labor and capital into output via the production functions

$$Q_i = \omega_i \, l_i^{\beta_l} k_i^{\beta_k}, \;\; i = 1, 2$$

with ω_i denoting plant-level technical efficiency, $\omega_1 > \omega_2$, and $\beta_l + \beta_k < 1$. At the output-maximizing allocation of labor $(l_1^*, k_1^*, l_2^*, k_2^*)$ marginal products are equated across plants for each input x

$$\frac{\partial Q_1(l_1^*, k_2^*)}{\partial x} = \frac{\partial Q_2(l_2^*, k_2^*)}{\partial x} \;\;\; x = l, k,$$

with plant 1 using more inputs in equilibrium than plant 2.

Now suppose that wedges similar to those in Hsieh and Klenow (2009) exist, where the wedges - whatever economic distortion may be causing them - can be represented by the more productive plant 1's output being subsidized at rate τ_1 and plant 2's output being taxed at rate τ_2. Wages and rental rates are assumed to be fixed. Plant 1 will use too many inputs and plant 2 will use too few. Let $(l_i^* + \Delta l_i(\tau_i), k_i^* + \Delta k_i(\tau_i))$ represent the distorted input levels of labor and capital at plant i. If these economic distortions are removed then the resulting change in plant i's output is given by integrating over the marginal products

$$\Delta Q_i = \int_{l_i^* + \Delta l_i(\tau_i)}^{l_i^*} \int_{k_i^* + \Delta k_i(\tau_i)}^{k_{*i}} \frac{\partial Q_i^2(l, k)}{\partial l \partial k} \, dk \, dl,$$

Aggregate output increases by $\Delta Q_1 + \Delta Q_2$, which is the difference between the gained output from plant 2 and the lost output from plant 1.

What does Olley-Pakes report as aggregate productivity growth from reallocation? If we let Δs_i denote the change in output share and $\bar{\omega} = \frac{\omega_1+\omega_2}{2}$ then Olley-Pakes reallocation is defined as

$$\Delta s_1(\omega_1 - \bar{\omega}) + \Delta s_2(\omega_2 - \bar{\omega}) < 0,$$

so despite output increasing due to the removal of the wedges, Olley-Pakes-measured reallocation decreases because $\Delta s_1 < 0, (\omega_1 - \bar{\omega}) > 0, \Delta s_2 > 0$, and $(\omega_2 - \bar{\omega}) < 0$. The reason is that average products and marginal products are negatively correlated. Overall Olley-Pakes aggregate productivity growth decreases because OP technical efficiency change is equal to zero: $\bar{\omega} - \bar{\omega} = 0$, and the sum of OP technical efficiency and OP reallocation equals OP total aggregate productivity growth.[2]

2 Aggregate Productivity Growth and Reallocation

We start by illustrating the Petrin and Levinsohn (2012) decomposition of aggregate productivity growth (APG) in a setting with no intermediate inputs or capital. In Section 2.2 we generalize the setup. In both cases APG is defined such that aggregation of plant-level changes in technical efficiency and input reallocations add up to changes in final demand, holding capital and labor use constant.

2.1 One-input Economy

There are N plants in the economy each producing a single good with a single input labor l. Production technologies are given by

$$Q_i(l_i, \omega_i),$$

with ω_i denoting the level of plant i's technical efficiency. With no intermediate inputs total output at plant i that goes to final demand is just Q_i. Assuming a common wage W and letting P_i denote the price of plant i's output APG is then given as the difference between the change in aggregate

[2]Note that the average share both before and after the wedges are removed is equal to $1/2$ and technical efficiencies do not change, so those terms just difference out.

final demand and the change in aggregate costs:

$$APG \equiv \sum_i P_i dQ_i - \sum_i W dl_i, \tag{2}$$

By totally differentiating $Q_i(l_i, \omega_i)$ one can see that (2) decomposes as:

$$\sum_i (P_i \frac{\partial Q_i}{\partial l} - W) dl_i + \sum_i P_i \frac{\partial Q_i}{\partial \omega_i} d\omega_i. \tag{3}$$

$\sum_i P_i \frac{\partial Q_i}{\partial \omega_i} d\omega_i$ are the total gains from technical efficiency changes and are equal to the sum over i of the value of the extra output firm i is able to produce given $d\omega_i$. Reallocation growth is given by

$$\sum_i (P_i \frac{\partial Q_i}{\partial l} - W) dl_i$$

so if dl_i of labor that was previously unemployed is reallocated to plant i then the value of aggregate output changes by $(P_i \frac{\partial Q_i}{\partial l} - W)$, the difference between the value of the marginal product and the input price. In the case where a small amount of labor reallocates from j to i so $dl_i = -dl_j$ aggregate output would change by the difference in the value of marginal products between i and j:

$$P_i \frac{\partial Q_i}{\partial l} - P_j \frac{\partial Q_j}{\partial l}.$$

In the case that labor reallocates across plants but total labor is held constant ($\sum_i dl_i = 0$), the change in aggregate output from reallocation is given by

$$\sum_i P_i \frac{\partial Q_i}{\partial l} dl_i.$$

2.2 General Setup

The production technology is now given by $Q_i(X_i, M_i, \omega_i)$, where $X_i = (X_{i1}, \ldots, X_{iK})$ is the vector of K primary input amounts (types of labor and capital) used at plant i and $M_i = (M_{i1}, \ldots, M_{iJ})$

is the vector giving the amount of each plant j's output used as an intermediate input at plant i.[3] The total amount of output from plant i that goes to final demand Y_i is then

$$Y_i = Q_i - \sum_j M_{ji},$$

where $\sum_j M_{ji}$ is the total amount of i's output that serves as intermediate input within plant i and across other plants $j \neq i$. The amount of i's output that goes to final demand is then given as $dY_i = dQ_i - \sum_j dM_{ij}$. APG is again given as the difference between the change in aggregate final demand and the change in aggregate costs, and in this generalized setup is equal to:

$$APG \equiv \sum_i P_i dY_i - \sum_i \sum_k W_{ik} dX_{ik}, \tag{4}$$

where W_{ik} equals the unit cost to i of the k^{th} primary input and dX_{ik} is the change in the use of that primary input at plant i.[4]

(4) decomposes as:

$$\sum_i \sum_k (P_i \frac{\partial Q_i}{\partial X_k} - W_{ik}) dX_{ik} + \sum_i \sum_j (P_i \frac{\partial Q_i}{\partial M_j} - P_j) dM_{ij} + \sum_i P_i \frac{\partial Q_i}{\partial \omega_i} d\omega_i, \tag{5}$$

where $\frac{\partial Q_i}{\partial X_k}$ and $\frac{\partial Q_i}{\partial M_j}$ are the partial derivatives of the output production function with respect to the kth primary input and the jth intermediate input respectively, dM_{ij} is the change in intermediate input j at plant i. $\sum_i P_i \frac{\partial Q_i}{\partial \omega_i} d\omega_i$ is again the gain from technical efficiency changes and reallocation is now given as

$$\sum_i \sum_k (P_i \frac{\partial Q_i}{\partial X_k} - W_{ik}) dX_{ik} + \sum_i \sum_j (P_i \frac{\partial Q_i}{\partial M_j} - P_j) dM_{ij}.$$

where the reallocation terms include a value of marginal product term and an input cost term for each plant and every primary and intermediate input. We now turn to estimation.

[3] Here we suppress their fixed cost term for transparency.

[4] In the general setup from Petrin and Levinsohn (2012) the path of primary and intermediate inputs and productivity shocks for plant i is given as $Z_{it} = (X_{it}, M_{it}, \omega_{it})$, $t \in [0, 1]$. For the entire economy they write $Z_t = (Z_{1t}, Z_{2t}, \ldots, Z_{Nt})$. Given Z_t, output quantities are determined by the production technologies and $Q_t = (Q_{1t}(Z_{1t}), \ldots, Q_{Nt}(Z_{Nt}))$. Prices are assumed to be uniquely determined by Q_t, given as $P_t = (P_{1t}(Q_t), \ldots, P_{Nt}(Q_t))$, and similarly for primary input costs $W_t = (W_{1t}(Z_t), \ldots, W_{Kt}(Z_t)))$. Y_{it} can then be directly calculated for all i and $t \in [0, 1]$.

2.3 Estimation

In growth rates APG can be expressed as the weighted sum of establishment-level growth rates in value added minus the establishment-level growth rates in primary inputs and is given as

$$APG = \sum_i D_i^v dlnVA_i - \sum_i \sum_k s_{ik}^v dlnX_{ik}, \tag{6}$$

with $D_i^v = \frac{VA_i}{\sum_i VA_i}$(the value-added Domar weight) and the cost share for the kth primary input given as $s_{ik}^v = \frac{W_{ik}X_{ik}}{\sum_i VA_i}$. For estimation we work with both gross output and value added production functions. We write the gross output production function as

$$ln(GO_i) = \sum_k \varepsilon_{ik} lnX_{ik} + \sum_j \varepsilon_{ij} dlnM_{ij} + ln\omega_i, \tag{7}$$

with ε_{ik} and ε_{ij} denoting the elasticities of gross output with respect to primary and intermediate inputs, respectively. Establishment-level gross output technical efficiency is given as $ln\omega_i$. APG can then be decomposed as

$$\underbrace{\sum_i D_i \sum_k (\varepsilon_{ik} - s_{ik}) dlnX_{ik}}_{\text{Reallocation of Labor and Capital}} + \underbrace{\sum_i D_i \sum_j (\varepsilon_{ij} - s_{ij}) dlnM_{ij}}_{\text{Reallocation of Intermediates}} + \underbrace{\sum_i D_i dln\omega_i}_{\text{Technical Efficiency}} . \tag{8}$$

where $D_i = \frac{P_i Q_i}{\sum_i VA_i}$ are gross output Domar weights and $s_{ik} = \frac{P_{ik}X_{ik}}{P_i Q_i}$ and $s_{ij} = \frac{P_{ij}M_{ij}}{P_i Q_i}$ are output shares for primary and intermediate inputs. Aggregate growth arising from the reallocation of primary inputs and intermediates inputs are given by $\sum_i D_i \sum_k (\varepsilon_{ik} - s_{ik}) dlnX_{ik}$ and $\sum_i D_i \sum_k (\varepsilon_{ij} - s_{ij}) dlnM_{ij}$, respectively. Growth from aggregate technical efficiency is given by $\sum_i D_i dln\omega_i$. We write valued added production functions as

$$ln(VA_i) = \sum_k \varepsilon_{ik}^v lnX_{ik} + ln\omega_i^v, \tag{9}$$

with ε_{ik}^v denoting the elasticity of (value-added) output with respect to the primary inputs, and the establishment-level value-added technical efficiency given as $ln\omega_i^v$. In this case APG can then be decomposed as

$$\underbrace{\sum_i D_i^v \sum_k (\varepsilon_{ik}^v - s_{ik}) dlnX_{ik}}_{\text{Reallocation of Labor and Capital}} + \underbrace{\sum_i D_i^v dln\omega_i^v}_{\text{Technical Efficiency}} \quad . \tag{10}$$

Aggregate growth arising from the reallocation of primary inputs is given by $\sum_i D_i^v \sum_k (\varepsilon_{ik}^v - s_{ik}) dlnX_{ik}$ and growth from aggregate technical efficiency is given by $\sum_i D_i^v dln\omega_i^v$. In equation (10) any growth from reallocation of intermediates will be incorrectly measured as growth from aggregate technical efficiency.

Equation (6) can be estimated directly from discrete-time data using Tornquist-Divisia approximations.[5] We estimate production function parameters in equation (7) separately for each SIC 4-digit industry for U.S. manufacturing, for each SIC 3-digit industry for Chile and Colombia, and NACE 2-digit industry code for Slovenia using the proxy method from Wooldridge (2009) that modifies Levinsohn and Petrin (2003) to address the simultaneous determination of inputs and productivity.[6] In the gross output case, the estimate of plant-level technical efficiency is

$$\widehat{ln\omega}_{it} = ln(GO_{it}) - \left(\widehat{\epsilon}_{jP}\, lnL_{it}^P + \widehat{\epsilon}_{jNP}\, lnL_{it}^{NP} + \widehat{\epsilon}_{jK}\, lnK_{it} + \widehat{\epsilon}_{jE}\, lnE_{it} + \widehat{\epsilon}_{jM}\, lnM_{it}\right),$$

where $\widehat{\epsilon}_{j\cdot}$ denote the estimated elasticities of gross output with respect to the inputs in industry j. We use Tornquist-Divisia approximations for each term in equation (8).[7] As regressors, we use three primary inputs and two intermediate inputs: production (blue-collar) workers L_{it}^P, non-production (white-collar) workers L_{it}^{NP}, capital K_{it} , energy E_{it} , and materials M_{it} .

In the value added case, the estimate of establishment-level technical efficiency is

$$\widehat{ln\omega^v}_{it} = ln(VA_{it}) - \left(\widehat{\epsilon^v}_{jP}\, lnL_{it}^P + \widehat{\epsilon^v}_{jNP}\, lnL_{it}^{NP} + \widehat{\epsilon^v}_{jK}\, lnK_{it}\right),$$

where $\widehat{\epsilon^v}_{j\cdot}$ denote the estimated elasticities of value added with respect to the inputs in industry j.

[5] We chain-weight to update prices on an annual basis (they are included in the Domar weights). For example, $APG = \sum_i \overline{D}_{it}^v \Delta lnVA_{it} - \sum_i \overline{D^v}_{it} \sum_k \overline{s}_{ikt}^v \Delta lnX_{ikt}$ where $\overline{D}_{it}^v$ is the average of establishment i's value-added share weights from period $t-1$ to period t, Δ is the first difference operator from period $t-1$ to period t, $\overline{s}_{ikt}$ is the average across the two periods of establishment i's expenditures for the kth primary input as a share of establishment-level value-added.

[6] The approach is robust to the comment by Ackerberg, Caves, and Frazer (2008) and is one line of code in Stata.

[7] For the reallocation terms we use the approximations $\sum_i \overline{D}_{it} \sum_k (\varepsilon_{ik} - \overline{s}_{ikt}) \Delta lnX_{ikt}$ and $\sum_i \overline{D}_{it} \sum_j (\varepsilon_{ij} - \overline{s}_{ijt}) \Delta lnM_{ijt}$. For the technical efficiency term we use $\sum_i \overline{D}_{it} \Delta ln\omega_{it}$.

3 BHC/OP Aggregate Productivity Growth

The Bailey-Hulten-Campbell/Olley-Pakes type of indexes of aggregate productivity growth use a definition of productivity growth that is based on looking at the change in share-weighted average products in the industry:

$$\sum_i s_{it}\omega_{it} - \sum_i s_{it-1}\omega_{it-1} \tag{11}$$

where ω_{it} is the estimated technical efficiency residual at plant i at time t. The Bailey-Hulten-Campbell (BHC) decomposition was not intended to map micro-level changes in technical efficiency to their impact on aggregate output (indeed Hulten (1978) is the important reference on that topic). Thus whatever decomposition of this index one uses, whether it be Olley-Pakes (1996), BHC, Foster, Haltiwanger and Krizan (2001), or Griliches and Regev (1995), one is getting a decomposition of a quantity that does not add up to the actual change in industry value added holding inputs constant (i.e., industry aggregate productivity change).

3.1 OP/BHC Decompositions

Sivadasan (2009) and Harrison, Martin, and Nataraj (2011) focus on both OP and BHC decompositions. Our empirical results will focus on the Olley-Pakes decomposition of equation (11) but the discussion applies equally to the BHC decomposition.

The classic OP decomposition of $\sum_i s_{it}\omega_{it} - \sum_i s_{it-1}\omega_{it-1}$ is given as

$$\Delta\bar{\omega_t} + [\sum(s_{it} - \bar{s}_t)(\omega_{it} - \bar{\omega_t}) - \sum(s_{it-1} - \bar{s}_{t-1})(\omega_{it-1} - \bar{\omega_{t-1}})]. \tag{12}$$

The first term is the change in the *unweighted* averages of technical efficiency at time t minus the change in *unweighted* averages of technical efficiency at time t-1 and is referred to as the "technical efficiency" term. On measuring the contribution of increases in plant level technical efficiency to aggregate output there is an old literature on how one gets from micro-level data back to an "aggregate Solow residual" when firms differ in their technologies and/or technical efficiency levels (Solow (1956), Domar (1962), Hulten (1978), Basu-Fernald (2002), Petrin-Levinsohn (2012)). The result with micro-level data says that the change in technical efficiency should be weighted by the ratio of plant-level gross-output to industry value-added - the Domar weight - regardless of the technology.

The second term in equation (12) is the “aggregate reallocation” term. It tracks market share movements across plants with differing average products. Economic theory says that the contribution to industry output from the reallocation of inputs should be measured using the differences in inputs’ marginal products. Theory does not provide any reason to believe that marginal products and average products are equal to one another in a general economics setting, let alone positively correlated with one another. For example, consider an equilibrium with firms facing common input prices but having differing levels of technical efficiency (different average products). While the firms will use different amounts of inputs in equilibrium, *the marginal products for any one type of input will be equal across firms even though they have different average products.*

4 Data

This section describes our plant-level manufacturing data from the U.S., Chile, and Colombia, and firm-level data from Slovenia.

U.S. Manufacturing Data For the U.S. we use plant-level data from the Census Bureau’s Annual Surveys of Manufactures from 1976-1996. To construct our variables, we follow the detailed description in the data appendix of Petrin, White and Reiter (2011). Here we provide a brief description of the variables. For labor we observe production worker hours and production worker wages, the average number of production workers, total employment, and total salaries and wages. For capital, we observe book values of assets and capital expenditures. We use use industry deflators and depreciation rates from the BEA and the perpetual inventory method to construct capital stocks from these measures. Our measure of nominal gross output is the total value of shipments. For intermediate inputs we use measures of energy and materials. For energy we use the sum of the cost of fuels and purchased electricity. For materials inputs, we use the total cost of materials minus energy costs. Value add is gross output minus materials and energy. We use industry-level deflators from the NBER-CES Productivity Database to convert from nominal to real values.

Chilean and Colombian Manufacturing Data The Chilean and Colombian data are annual and span the periods of 1979-95 and 1977-91, respectively. Here we provide a brief overview of these data. Numerous other productivity studies use them, and we refer interested readers to those papers for a more detailed data description.[8]

The Chilean data, provided by Chile's Instituto Nacional de Estadistica (INE), are unbalanced panels and cover all manufacturing plants with at least 10 employees. The Colombian data from the Annual Manufacturing Survey, provided by Colombia's Departamento Administrativo Nacional de Estadistica (DANE), are also unbalanced panels and cover all plants for the years 1977-82 and the plants with at least 10 employees for the years 1983-91. In both data sets, plants are observed annually and they include a measure of nominal gross output, two types of labor, capital, and intermediate inputs, including fuels and electricity. Labor is the number of person-years hired for production, and plants distinguish between their blue- and white-collar workers. Liu (1991) documents the method for constructing the real value of capital for the Chilean data, and we use the same method for the Colombian data.[9] We use double-deflated value added for Chilean results and single-deflated value added for Colombia because intermediate input deflators are not available there.

Slovenian Manufacturing Data For Slovenian data, we use the annual accounting data provided by the Slovenian Statistical Office and other sources from 1994 through 2004. Our data are an unbalanced panel and covers all manufacturing firms.We use single-deflated value added because no intermediate input deflator is available. The Slovenian data are distinct from Chilean and Colombian data in that it is firm-level data and not plant-level data and there exists both a firm-level deflator and a capacity utilization rate for a subset of firms.

As an ex-socialist country Slovenia went through extensive changes in its economic system

[8] See Lui (1991), Lui (1993), and Levinsohn and Petrin (2003) for the Chilean data and Roberts (1996) for the Colombian data.

[9] For the Chilean data, the real value of capital is a weighted average of the peso value of depreciated buildings, machinery, and vehicles. We assume each has a depreciation rate of 5%, 10%, and 20%, respectively. Some plants don't report initial capital stock, although they record investment. When possible, we used a capital series that they report for a subsequent base year. For a small number of plants, they don't report capital stock in any year. We estimated a projected initial capital stock based on other reported plant observables for these plants. We then used the investment data to fill out the capital stock data.

starting in 1988. The deregulation of entry in 1988 allowed the setup of privately owned firms and resulted in expansion of private businesses. In addition, price and wage liberalization took place during the period of 1987-93. The process of privatization of state-owned firms started in 1994 and continued throughout the 1990s. For this reason, several empirical studies of productivity dynamics have used Slovenian data.[10]

5 Results

Figure 1 shows a graph of the annual growth rates of aggregate value-added and aggregate productivity (equation 4) for U.S. manufacturing from 1977-1996. The difference between the two is the sum of the growth rates of primary input costs.[11] Aggregate productivity grew 1.91 % per year. As the graph shows, aggregate productivity growth is highly correlated with the growth of value added–most of the fluctuations in aggregate productivity are primarily associated with fluctuations in value-added.

Figure 2 plots aggregate productivity growth (equation 4) along with the growth rate of the Olley-Pakes aggregate productivity index (equation 12).[12] In most years the two measures are quite different. The average OP measure of aggregate productivity growth is -1.57% per year (versus *positive* 1.91% per year for APG), and the correlation between the two series is only 0.51.

Figure 3 presents per annum averages of reallocation's contribution to aggregate productivity growth for each 5-year period from 1977 to 1996, using three different measures of reallocation: the Petrin-Levinsohn measure with gross output production functions (equation 8), the APG measure using value-added production functions (equation 10), and the Olley-Pakes measure of reallocation (equation 12). In each period, the APG measure of reallocation using gross output production functions is positive and larger than the APG measure using value-added production functions. In the final period (1992-1996), the gross output measure of the annual average contribution is 2.11%

[10]See, for example, Konings and de Loecker (2006), Polanec (2006), and Bartelsman, Haltiwanger, and Scarpetta (2010).

[11]Columns 1-5 of Table A1 in the appendix present the annual growth rates of, respectively, value-added, primary inputs costs (production worker labor, non-production worker labor, and capital), and aggregate productivity.

[12]Column 6 of Table A1 presents the annual growth rates for the Olley-Pakes index.

higher than the value-added measure of reallocation. The Averaging across all years, the APG gross output measure of reallocation contributes 2.15% per year, and it contributes positively in all but two years.[13] The APG gross output measure of technical efficiency growth contributes −0.54% per year. In contrast, when we estimate value-added production functions (equation 9) and aggregate using value-added share weights as in equation (10), technical efficiency growth contributes *positive* 0.77% per year, and reallocation (excluding intermediates) contributes only 1.13 per year. The average OP measure of aggregate reallocation's contribution to growth is *negative* 2.49 % per year. Thus in the U.S. data, the APG decomposition using the value-added specification understates the contribution of reallocation by 1.02% per year, and the OP measure understates reallocation's contribution by 4.64 percentage points per year

Table 1 presents the decomposition of the APG gross output measure of reallocation into the separate contributions of production worker and non-production worker labor, capital, and intermediate inputs. Reallocation of capital makes the largest contribution, but reallocation of intermediate inputs accounts for 0.85 percentage points per year — 40% of the total contribution of reallocation. This explains why the APG gross output measures of reallocation and technical efficiency growth are so different from the APG value added measures: in the value-added specification, the contribution of reallocation of intermediates is attributed to technical efficiency growth.

Table 2 summarizes the differences between the APG and OP measures of aggregate productivity growth for Chile, Colombia, and Slovenia. Using the APG measure, all three countries' manufacturing sectors experienced significant productivity growth over the respective sample periods: 3.40%, 2.93%, and 4.17% per year in Chile, Colombia, and Slovenia, respectively. In contrast, the OP measure of aggregate productivity growth is 0.14% per year for Chile, 1.79% for Colombia, and −0.03% for Slovenia.

Table 3 summarizes the differences between the APG and OP decompositions for Chile, Colombia, and Slovenia. Just as in the U.S. manufacturing sector, in both Chile and Colombia, the APG gross output measure of reallocation is significantly higher than the value-added measure–1.87 percentage points per year higher in Chile and 1.24 percentage points higher in Colombia. The OP

[13]Table A2 in the appendix presents the annual contributions of each measure of reallocation as well as the annual contributions of the corresponding measures of technical efficiency growth.

measure significantly understates the contribution of reallocation by 3.43, 1.64, and 1.91 percentage points per year in, respectively, Chile, Colombia, and Slovenia.

Table 4 summarizes the decomposition of the APG gross output measure of reallocation into the separate contributions of production worker and non-production worker labor, capital, and intermediate inputs for Chile, Colombia, and Slovenia. In all three countries, reallocation of intermediates makes a larger contribution to growth than reallocation of any other input: 1.26 percentage points per year in Chile, 2.52 in Colombia, and 0.35 in Slovenia. The relatively small contribution of intermediates reallocation in Slovenia (compared to the other countries) explains why the APG gross output and value-added measures of total reallocation are similar in that country (see table 3).

6 Conclusions and Suggestions for Future Research

Recent studies have found that reallocation of resources across plants played surprisingly little role in the large increase in aggregate productivity in India in recent years. These findings may be an artifact of the way these studies measure the contributions of technical efficiency growth and reallocation. Using data from 4 countries — the U.S., Chile, Colombia, and Slovenia — we show that ignoring reallocation of intermediate inputs significantly understates (overstates) the contribution of reallocation (technical-efficiency growth) in aggregate productivity growth. Furthermore, we show that in these four countries using average products instead of marginal products underestimates the contribution of reallocation. Of course, this does not necessarily imply that reallocation made a large contribution to aggregate productivity growth in the Indian manufacturing sector in recent years. Future research is needed to investigate the implications of these findings for India and other countries.

Table 1: Decomposition of Reallocation:
U.S. Manufacturing, 1977–1996

	Percentage Growth Rates from Reallocation of ...			
	(1)	(2)	(3)	(4)
Year	Production workers	Non-Production workers	Capital	Intermediate Inputs
1977	0.44	0.14	0.94	3.50
1978	0.43	0.05	1.21	2.29
1979	0.10	0.13	1.02	0.43
1980	-0.53	0.05	1.66	-2.05
1981	0.08	0.01	1.69	0.02
1982	-0.86	0.04	1.30	-3.78
1983	-0.05	0.03	1.12	0.48
1984	0.35	-0.04	0.84	4.01
1985	0.05	0.01	1.65	1.63
1986	0.05	0.06	1.36	0.99
1987	0.09	0.10	1.02	0.99
1988	0.37	0.06	0.88	1.59
1989	0.14	0.02	0.79	0.33
1990	-0.59	0.09	1.61	-0.60
1991	0.00	0.14	1.70	-1.43
1992	0.15	-0.08	0.84	1.14
1993	0.03	0.01	1.34	0.99
1994	0.10	0.01	0.79	1.90
1995	-0.07	0.15	1.31	1.57
1996	0.25	0.02	1.31	3.07
mean	0.03	0.05	1.22	0.85
s.d.	0.34	0.06	0.32	1.86

Gross Output Production Functions estimated by Wooldridge (2009) modification of Levinsohn and Petrin (2003) estimator.

**Table 2: Percentage Growth Rates Per Year,
Value-Added, Primary Input Costs and Aggregate Productivity,
Chilean, Colombian, and Slovenian Manufacturing.**

	(1)	(2)	(3)	(4)	(5)	(6)
					APG	OP
	Value	Production	Non-production	Capital	Aggregate	Aggregate
Country	Added	labor costs	labor costs	costs	Productivity	Productivity
Chile	4.01	0.42	0.28	-0.09	3.40	0.14
Colombia	3.55	0.40	0.16	0.06	2.93	1.79
Slovenia	7.32	1.14	0.85	1.15	4.17	-0.03

Note: (1) - (2) - (3) - (4) = (5)

Table 3: Aggregate Productivity Growth Decompositions
Petrin-Levinsohn vs. Olley-Pakes, Chilean, Colombian, and Slovenian Manufacturing
Average Annual Percentage Growth Rates

		Aggregate Productivity Growth Contributions from...					
Country	(1) APG Aggregate Productivity Growth	(2) APG Technical Efficiency (GO)	(3) APG Technical Efficiency (VA)	(4) OP Technical Efficiency	(5) APG Reallocation (GO)	(6) APG Reallocation (VA)	(7) OP Reallocation
Chile	3.40	-0.54	1.48	0.48	3.09	1.22	-0.34
Colombia	2.93	0.08	1.07	-0.11	3.54	2.30	1.90
Slovenia	4.17	2.43	2.38	-0.01	1.89	1.99	-0.02

See notes for Table A2.

Table 4: Decomposition of APG Reallocation: Chilean, Colombian, and Slovenian Manufacturing.

Country	Average Annual Percentage Growth Rates from Reallocation of ...			
	(1)	(2)	(3)	(4)
	Production workers	Non-Production workers	Capital	Intermediates
Chile	0.39	0.24	0.28	1.26
Colombia	0.57	0.05	2.21	2.52
Slovenia	0.34	0.22	-0.27	0.35

Gross Output Production Functions estimated by Wooldridge (2009) modification of Levinsohn and Petrin (2003) estimator.

Figure 1: Value Added and Aggregate Productivity Growth, U.S. Manufacturing

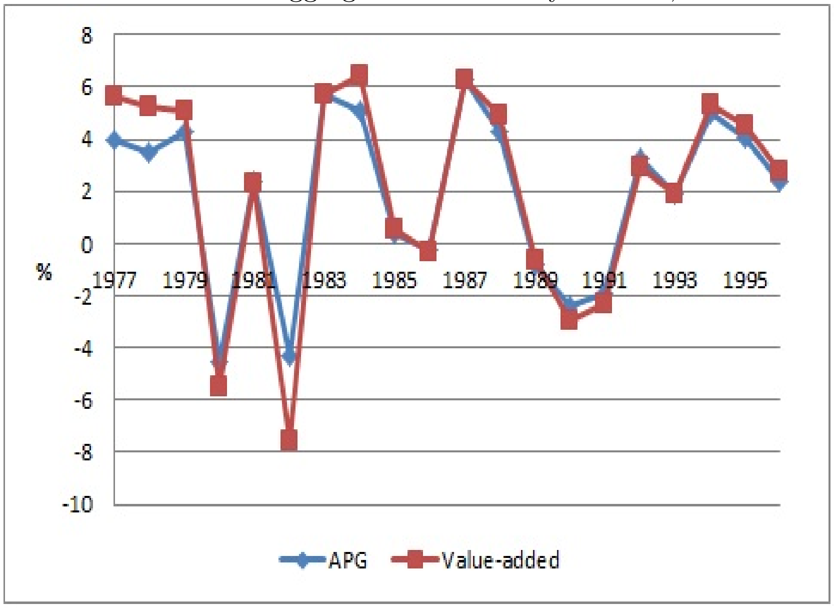

Figure 2: Petrin-Levinsohn APG vs. Olley-Pakes APG, U.S. Manufacturing

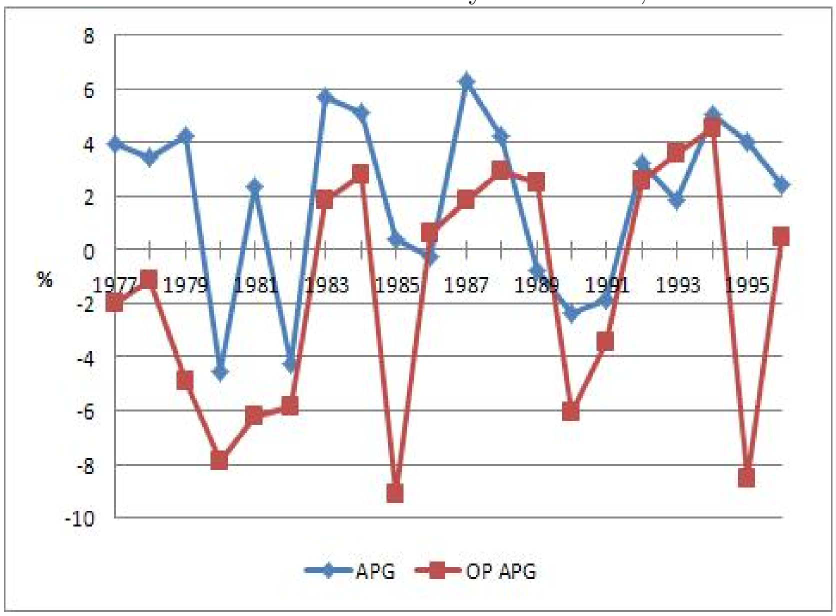

Figure 3: Aggregate Productivity Growth from Reallocation, Petrin-Levinsohn Gross Output vs. Value-Added vs. Olley-Pakes Decomposition U.S. Manufacturing, % Per Year

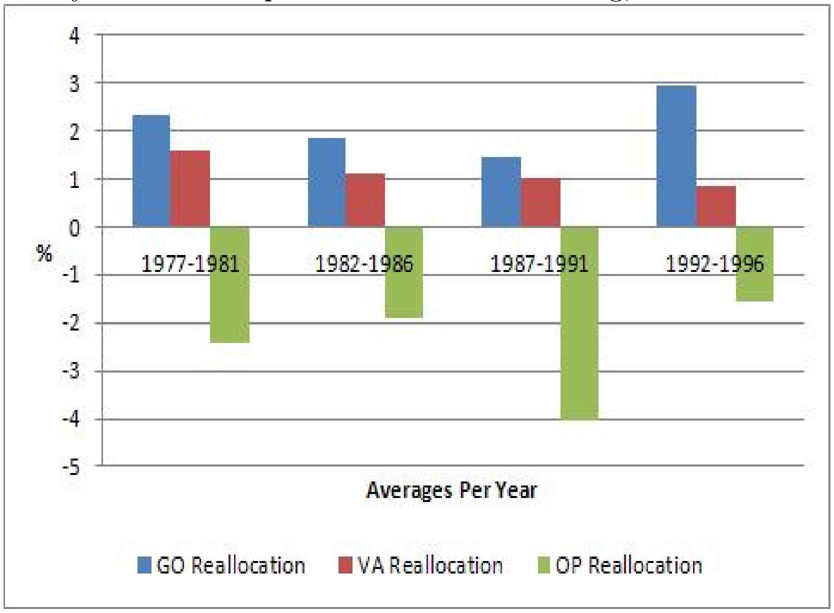

Table A1: Percentage Growth Rates of Value-Added, Primary Input Costs and Aggregate Productivity in U.S. Manufacturing, 1977–1996.

Year	(1) Value Added	(2) Production labor costs	(3) Non-production labor costs	(4) Capital costs	(5) APG APG	(6) OP APG
1977	5.66	1.03	0.41	0.29	3.94	-2.03
1978	5.23	0.88	0.50	0.39	3.46	-1.12
1979	5.06	0.02	0.43	0.35	4.27	-4.93
1980	-5.53	-1.96	0.60	0.40	-4.57	-7.91
1981	2.31	-0.51	0.02	0.45	2.34	-6.23
1982	-7.55	-3.41	-0.35	0.48	-4.27	-5.85
1983	5.74	0.02	-0.36	0.35	5.73	1.86
1984	6.45	1.05	0.18	0.11	5.12	2.80
1985	0.52	-0.51	0.32	0.34	0.37	-9.11
1986	-0.32	-0.56	0.11	0.37	-0.24	0.58
1987	6.27	0.01	-0.26	0.26	6.26	1.86
1988	4.95	0.37	0.06	0.24	4.28	2.92
1989	-0.68	-0.16	0.02	0.23	-0.77	2.52
1990	-2.96	-0.77	-0.22	0.42	-2.40	-6.08
1991	-2.35	-0.74	-0.09	0.38	-1.89	-3.46
1992	2.94	-0.04	-0.47	0.21	3.23	2.61
1993	1.89	0.05	-0.29	0.26	1.87	3.57
1994	5.35	0.31	-0.18	0.19	5.03	4.57
1995	4.50	0.09	0.08	0.31	4.01	-8.52
1996	2.76	0.05	-0.13	0.44	2.40	0.50
mean	2.01	-0.24	0.02	0.32	1.91	-1.57
s.d.	4.10	1.01	0.31	0.10	3.29	4.54

Note: (1) - (2) - (3) - (4)= (5)

Table A2: Aggregate Productivity Growth Decompositions
Petrin-Levinsohn vs. Olley-Pakes, U.S. Manufacturing 1977–1996

		Aggregate Productivity Growth Contributions from...					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	APG	APG	APG	OP			
	Aggregate	Technical	Technical	Technical	APG	APG	OP
	Productivity	Efficiency	Efficiency	Efficiency	Reallocation	Reallocation	Reallocation
Year	Growth	(GO)	(VA)		(GO)	(VA)	
1977	3.94	-2.25	2.51	-0.48	5.01	1.42	-1.55
1978	3.46	-1.66	1.46	-0.57	3.98	2.01	-0.56
1979	4.27	1.11	2.80	-5.6	1.69	1.46	0.67
1980	-4.57	-3.54	-5.98	-2.75	-0.88	1.42	-5.16
1981	2.34	-1.11	0.75	-0.59	1.80	1.59	-5.64
1982	-4.27	-1.66	-4.60	-1.59	-3.30	0.35	-4.25
1983	5.73	3.33	4.85	2.09	1.58	0.87	-0.23
1984	5.12	-0.30	3.85	4.89	5.16	1.25	-2.09
1985	0.37	-3.69	-1.21	-4.53	3.34	1.60	-4.58
1986	-0.24	-3.10	-1.64	-1.11	2.46	1.39	1.69
1987	6.26	3.44	5.07	5.85	2.21	1.18	-3.98
1988	4.28	1.26	3.00	-1.17	2.90	1.27	4.09
1989	-0.77	-2.18	-1.65	8.34	1.29	0.87	-5.82
1990	-2.40	-2.33	-3.23	5.38	0.51	0.84	-11.47
1991	-1.89	-1.63	-2.77	-0.33	0.41	0.88	-3.13
1992	3.23	1.45	2.53	4.61	2.04	0.69	-2.00
1993	1.87	-0.24	0.91	-2.32	2.38	0.95	5.88
1994	5.03	2.56	4.51	2.96	2.80	0.52	1.61
1995	4.01	1.35	2.97	6.75	2.96	1.03	-15.27
1996	2.40	-1.67	1.32	-1.43	4.64	1.08	1.93
mean	1.91	-0.54	0.77	0.92	2.15	1.13	-2.49
s.d.	3.29	2.22	3.23	3.93	2.00	0.40	4.97

Column 1 is the Petrin-Levinsohn (APG) (2012) measure of aggregate productivity growth. Columns 2-3 are APG technical efficiency growth using, respectively, gross output production functions and value-added production functions, respectively. Column 4 shows the Olley-Pakes measure of technical efficiency growth. Columns 5-7 show measures of reallocation's contribution to aggregate productivity growth using, respectively, the APG decomposition with gross output and value-added production functions and the Olley-Pakes decomposition with gross output production functions. Production functions are estimated by Wooldridge (2009) modification of Levinsohn and Petrin (2003) estimator. Columns 2 and 5 do not sum exactly to column 1 in part because of approximation error. Each column is approximating a continuous-time measure of growth using discrete-time data.

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